Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

April 1948 Vol. 1, No. 4

General Theoretical and Experimental Methods

(See also Revs. 602, 608)

576. **V. Rojansky, "Gyrograms for simple harmonic systems subjected to external forces,"** *J. appl. Phys.*, Mar. 1948, vol. 19, pp. 297–301.

A graphical method is described for the approximate integration of equations of the form $m\,d^2x/dt^2 + kx = f(t)$. The graphs, named "gyrograms," are an extension of the familiar "reference circle" used in representing simple harmonic motion. The method is applicable whenever $f(t)$ can be adequately approximated by a step function having a few steps. Each step provides a circular arc of the gyrogram. A numerical illustration is presented. Lester L. Cronvich, USA

General Dynamics, Kinematics, Friction

(See also Revs. 690, 707, 720, 735)

577. **R. L. Johnson, M. A. Swikert, and E. E. Bisson, "Friction at high sliding velocities,"** *Nat. adv. Comm. Aero. Tech. Note No. 1442*, Oct. 1947, pp. 1–35.

Experimental work is reported on the coefficient of kinetic friction μ_k of a steel ball sliding on a steel disk. Sliding speeds extended into the hitherto unexplored region of 1100–6600 fpm.

For dry rubbing surfaces μ_k remained constant (at about 0.54) up to a speed of around 2000 fpm. Beyond this speed μ_k decreased continuously and at the highest speeds investigated μ_k tended to approach an again constant value (roughly 0.2). Chemical changes, such as formation of an oxide, and physical changes of the surfaces were considered to offer a reasonable explanation for this trend.

For boundary-lubricated surfaces, μ_k decreased continuously with increasing speeds, from an initial value of 0.1 downward. For both dry and boundary-lubricated surfaces, μ_k was independent of load within the load range investigated.

The preparation of the rubbing surfaces was considered to be a principal variable, and much attention was devoted to it.

H. Blok, Holland

578. **M. L. Bykhovsky, "On a method of determining velocity and acceleration errors of plane mechanisms"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Ak. Nauk SSSR Ser. tekh. Nauk)*, May 1947, no. 5, pp. 503–510.

The paper presents general vectorial expressions for the errors in velocity and acceleration of the driven link of a plane mechanism, in terms of the errors of independent parameters determining the dimensions of all links of the mechanism and the positions of the leading links. The component parts of the total errors are due to the following causes: the deviation from the ideal scheme of the mechanism, the primary errors in the parameters, the errors in the velocities of the leading links, and the time variation of the primary errors. Determination of the component parts of the total error is discussed in detail and is demonstrated graphically, both in relation to the velocity and the acceleration of the driven link, on an example of a somewhat elementary mechanism consisting of a crank, a connecting rod, and a slider. A. Hrennikoff, Canada

579. **E. A. Chudakoff, "Influence of lateral elasticity of wheels on automobile control"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Ak. Nauk SSSR Ser. tekh. Nauk)*, Oct. 1947, no. 10, pp. 1287–1304.

The motorcar wheel which has lateral elasticity does not roll in a plane normal to its axis, but in a plane inclined to it at a certain angle. This is because wheels of the moving car are under the action of lateral forces.

The author deduces the equations of motion of an automobile when it is turning, and the lateral elasticity of the wheels is taken into consideration. From these equations he considers the influence of the elasticity of the wheels on car steering, and presents diagrams showing the results. These results indicate that lateral elasticity of motorcar wheels may complicate car control a great deal. Besides, it may also have an important influence on the lateral stability of an automobile, according to the author.

Witold Wierzbicki, Poland

Gyroscopics, Governors, Servomechanisms

580. **B. W. Millington, "Centrifugal governors with flyweights of distributed mass,"** *Engineering*, Mar. 28, 1947, vol. 163, p. 232.

This paper points out that it is not proper to consider that the mass of a flyweight governor is concentrated at the center of gravity. A distributed mass produces a centrifugal moment equal to the moment calculated on the assumption that the mass is concentrated at the center of gravity, together with a moment term involving the product of inertia of the flyweight.

Herbert Harris, Jr., USA

581. **H. L. Price, "Turning errors of a monitored directional gyroscope,"** *Aircr. Engng.*, Jan. 1948, vol. 20, pp. 11–17; Feb. 1948, vol. 20, pp. 38–45.

This is a thorough analysis of the nature and magnitude of the errors (introduced upon resumed straight flight) effected by a monitored directional gyroscope (DG) during the course of a true, banked, horizontal turn, with consideration of the influence of residual error on a coupled air position indicator (API). The purpose of the paper is to suggest precautions to be taken to minimize the resulting errors, but this reviewer believes it is also useful as a guide to constructive ideas on the difficult problem of eliminating the errors.

The investigation assumes a mechanically and electrically perfect instrument, and is confined to consideration of errors

arising from the aspects assumed by the gyroscopic system while subjected to the kinematics of turning and to applied monitoring and leveling torques. The analysis is applied to a DG employing a flux valve sensitive element. It covers static flux valve error; the generalized gimbal error; and the investigation of the actual motion of the gyroscopic rotor under the action of flux valve monitoring and leveling torques. Calculations cover reading error at any instant of turn and upon coming out of a turn; rotor leveling error, including that at the instant of recovery; and the time for monitoring out residual compass error. The influences of various individual parameters of the problem are assessed in respect to their error-making tendencies.

Errors are determined, including those in the coupled API, for both general and special cases. Formulas are given for gimbal error, and monitoring and leveling torque errors during turn (for a monitored DG only), for any monitoring and leveling torque forms. Charts are developed giving errors for a comprehensive range of parameters, including aircraft speed and rate of turn, monitoring, and leveling.

It is concluded that during a turn, monitoring and leveling torques may function to increase errors, but upon recovery, while there are residual errors from monitoring and leveling torques, gimbal errors disappear. Upon recovery from a turn, a residual error produces an increment of error in the API. The values of residual errors depend upon the aircraft speed, rate of turn and angle of bank, the direction in space of the rotor axis, and the rates of monitoring and leveling. Residual error is minimized if the rotor axis points N-S or E-W. At a given bank and monitoring rate, the air position indicator error increases with the speed, in a manner depending upon the type of monitoring. The ultimate conclusion is that the most effective error reduction is accomplished by eliminating both monitoring and leveling during a turn.

Complete explanatory diagrams for the navigational factors related to the aircraft and DG are included.

Lloyd R. Koenig, USA

Vibrations, Balancing

(See also Revs. 576, 579, 661, 721)

582. Dana Young, "Vibration of a beam with concentrated mass, spring, and dashpot," *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 65–72.

An analytical method is shown for determining the natural frequencies of a composite system consisting of a uniform beam with concentrated masses, springs, and dashpots attached at any point along the length of the beam. The author considers only a beam of constant cross section, and rotary inertia and the effect of shear are neglected. The spring which acts on the concentrated mass is assumed to have negligible mass. Most of the details of the analysis are carried out for a cantilever beam, but the procedure is quite general.

The equation of motion for a beam subjected to a distributed harmonic load $f(x) \sin \omega t$ is

$$EI \frac{\partial^4 y}{\partial x^4} + \frac{m}{l} \frac{\partial^2 y}{\partial t^2} = f(x) \sin \omega t$$

where m and l are the mass and the length. This is integrated by the use of a series expansion in terms of a set of orthogonal functions which represent the normal modes of vibrations of the beam alone. Numerical examples are given for a cantilever beam with one and two concentrated masses, springs, and dashpots.

J. Kozešník, Czechoslovakia

583. R. B. Meuser and E. E. Weibel, "Vibration of a nonlinear system during acceleration through resonance," *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 21–24.

An investigation of the increase in vibration amplitude during acceleration of a machine through a critical speed is of interest for the theorist as well as from the practical point of view. This article describes an experimental investigation of this process, made with a spring mass system with a single degree of freedom. The harmonic exciting force was of constant amplitude and its frequency increased uniformly with the time. The spring force did not vary linearly with the displacement x but was of the type $k_1x + k_2x^3$. The damping of the system was proportional to the velocity.

The results of the experiments are graphically shown as relations of dimensionless parameters, and for the range of the investigation permit the following conclusions: The maximum amplitudes of vibration of the mass are reduced if the spring constant k_2 is increased. Exceptions to this rule were observed only for very small damping. On the other hand the maximum amplitudes of the spring force increase with increase in the spring constant k_2. These results can easily be applied to the starting up of reciprocating engines which are coupled to electric motors or generators.

Pavel Kohn, Czechoslovakia

584. F. H. Todd, "The fundamentals of ship vibration," *J. Amer. Soc. nav. Engrs.*, Feb. 1948, vol. 60, pp. 86–110.

This paper shows that the natural frequencies of a ship may be calculated as a free-free beam, provided the mass of the beam is increased by an "added virtual mass" to account for energy dissipated in the water. The vibration amplitude is generally small except where amplified by structural resonance. The paper discusses methods of reducing the magnitude of disturbing forces and avoiding resonant conditions. Charles E. Crede, USA

585. M. S. Anziferov, "Free transverse vibrations of a rod with movably clamped ends" (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), Dec. 1947, vol. 17, no. 12, pp. 1451–1456.

The author considers the lateral vibrations of a bar which at one end satisfies the boundary conditions $X' = 0$, $X''' = 0$ and at the other is clamped, $X = 0$, $X' = 0$ (where $X(x)$ is the lateral deflection). This and two other variations of this problem are solved, and in each case the first six characteristic roots are given. A possible experimental realization of the boundary condition $X' = 0, X''' = 0$ is given.

J. V. Wehausen, USA

Wave Motion, Impact, Seismology

(See also Revs. 587, 597, 703, 709, 731, 737)

586. J. I. Sekerzh-Zenkovich, "On the theory of stationary waves of finite amplitude on the surface of a heavy fluid" (in Russian), *Notes Acad. Sci. USSR* (*Doklady Ak. Nauk SSSR*), Nov. 1, 1947, vol. 58, no. 4, pp. 551–553.

In order to investigate the exact form of the waves mentioned in the title, the author, using Lagrangian co-ordinates, introduces a parameter into the functions describing the motion of the particles. These functions, which must satisfy appropriate differential equations and boundary conditions, are holomorphic in the parameter and uniquely determined for sufficiently small values of the parameter. Also, a velocity potential exists for the flow. The author next expands his functions in powers of the parameter, keeping terms through the fourth power. From this

he is able to obtain approximately the shape of the surface wave, which he finds to be close to a trochoid. He also finds that there are no truly fixed nodes, the nodal points oscillating back and forth. Proofs for these statements will presumably be given in a more detailed presentation. J. V. Wehausen, USA

Acoustics

587. R. B. Lindsay, "Compressional wave-front propagation through a simple vortex," *J. acoust. Soc. Amer.*, Mar. 1948, vol. 20, pp. 89–94.

The problem considered is the propagation of sound in a medium containing a straight vortex. The exact formal solution involves elliptic functions; approximate solutions, valid for rays which do not pass too near the axis of the vortex, are therefore developed. The main result may be expressed by the statement that the phase difference for any two points is altered by an amount proportional to the vortex strength, and to the angular separation of the points relative to the vortex—the angle varying continuously along each ray. Estimates of the order of magnitude of the phase changes to be expected from this cause due to atmospheric eddies agree with the phase fluctuations which have been observed. W. G. Bickley, England

588. E. L. Carstensen and L. L. Foldy, "Propagation of sound through a liquid containing bubbles," *J. acoust. Soc. Amer.*, May 1947, vol. 19, pp. 481–501.

The acoustic transmission and reflection of a screen of air bubbles in water were tested experimentally. The original purpose was to give information about the acoustic properties of marine wakes. The frequency range was 5-45 kc per sec and measurements were made in conventional fashion with projector and hydrophones. The bubbles were produced by nozzles, each with a central air jet and a sheath of flowing water around it. A continuous-flow screen could be used with bubble sizes ranging between 0.1 and 1 mm radius. Alternatively, a pulsed screen could be formed with fewer bubbles, all of uniform size. The size, distribution, number, and rate of rise of the bubbles were investigated in detail.

The theory of acoustic attenuation in and reflection from a screen of bubbles is reviewed, and is then applied to the particular experimental circumstances. Equations are presented for the attenuation and reflection coefficients to be expected as a function of frequency, for both continuous-flow and pulsed screens. The coefficients depend on the damping rate per bubble, and particularly on the value of this rate at bubble resonance. The resonance damping rates were evaluated from the experimental data. They increase roughly linearly with the frequency, as predicted by theory, but are two or three times larger than predicted values.

The curves of measured attenuation and reflection coefficients versus frequency agree quite well with theory for the pulsed screen, but not very well for continuous-flow screens. Possible reasons for the discrepancies are discussed. The practical conclusion was that bubble effects suffice to explain all the acoustic properties of wakes. A. O. Williams, Jr., USA

589. R. W. Young, "Image interference in the presence of refraction," *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 1–7.

This paper is a theoretical and experimental study of the so-called Lloyd mirror effect in the propagation of sound through water possessing a temperature gradient which produces refraction. A simplified theoretical deduction is given on the basis of equivalent straight-line rays for the resultant excess pressure produced in the water by the combination of the direct sound from an underwater source and the sound reflected from the water-air interface. Both upward and downward refraction are considered.

The theoretically predicted interference patterns are compared with the results of experimental measurements with a transducer mounted about 14 ft below the surface and emitting sound at frequencies of 0.2, 0.6, 1.8, 7.5, and 22.5 kc per sec. Good agreement was found at the first three frequencies for hydrophone depths of the order of 300 ft. In particular the contracted pattern for downward refraction and the extended pattern for upward refraction were clearly evident. No interference pattern was found for 22.5 kc per sec. The roughness of the sea surface seemed to have had little or no effect at 0.2 kc per sec, but nothing is said about its effect at higher frequencies. The effect of multiple surface reflection is not considered. R. B. Lindsay, USA

590. H. K. Schilling, M. P. Givens, W. L. Nyborg, W. A. Pielemeier, and H. A. Thorpe, "Ultrasonic propagation in open air," *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 222–234.

Measurements were made of the transmission of sound (at frequencies of 10 to 30 kc per sec) over open terrain in the tropics and temperate zones. At times the decrease of intensity with distance, over and above the value to be expected from the inverse-square law, is exponential, although complications arise from thermal gradients, winds, etc. At other times the air acts in addition as a scattering medium. This is attributed to the existence, at times, of a very complex "micrometeorological" structure of the air near the ground.

The effect of these variables on the possibilities of ultrasonic signaling in the open air is discussed. It seems likely that reliable signaling over distances exceeding a few hundred feet is not possible. Martin Greenspan, USA

591. A. M. Hill, "Underwater acoustical measuring facilities at the U. S. Navy underwater sound laboratory," *J. acoust. Soc. Amer.*, Sept. 1947, vol. 19, pp. 907–909.

The author presents a brief description of the apparatus employed for measuring sound characteristics of high-frequency (sonar) transducers and receivers, as well as a short analysis of the procedure of making measurements of sound intensity. The whole assembly and the complete block diagram of the apparatus also appears in the paper.

No results of any experiments are presented, although it should be very interesting to see if the experimental data on sound intensity and directional and phase characteristics would agree with Stenzel's theory on the characteristics of a sound source under water [*Ann. Phys.*, 1943, vol. 43, p.1].

Nicholas Chako, USA

592. R. J. Urick and H. L. Saxton, "Surface reflection of short supersonic pulses in the ocean," *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 8–12.

An ultrasonic source at a depth of 200 ft in the open ocean emitted 0.5 millisecond pulses of 25 kc per sec sound with a repetition rate of 50 or 100 per sec. The pulses were picked up at 12-ft depth at ranges from 200 to 1400 yd, and displayed on an oscilloscope screen arranged to permit the separation of primary and secondary arrivals. Study of the time separation of the primary and secondary arrivals showed that the former are due to the sound traveling directly from the source to the receiver, the latter to reflections at the water surface.

The results indicate (1) that the surface reflections observed in

these experiments were mostly specular and not diffuse, and (2) that the reflection occurred with little loss in intensity. The sea surface is described merely as "choppy," and the water is said to have been isothermal to a depth of 300 ft.

R. B. Lindsay, USA

Elasticity Theory

(See also Revs. 602, 626, 633, 635)

593. E. L. Buell, "On the distribution of plane stress in a semi-infinite plate with partially stiffened edge," *J. Math. Phys.*, Jan. 1948, vol. 26, pp. 223–233.

The paper deals with the plane stresses induced in a semi-infinite flat plate having a stiffening rod rigidly attached to its straight edge; the stiffener extends to infinity in one direction only, from a point of the straight edge, and an axial load is applied to the stiffener at this point. Previous contributions along allied lines include the classic solution of J. Boussinesq for a semi-infinite plate loaded by a concentrated force applied at a point of its unstiffened straight edge, and the work of E. Melan for the analogous case where an edge stiffener which extends to infinity in both directions is added to the straight edge (see bibliography at end of paper).

To effect the solution, the boundary conditions are first established in terms of the function $\varphi(x, y)$ which is contained in the Airy stress function $\Phi = y\varphi(x, y)$, where $\varphi(x, y)$ is a potential function in terms of the Cartesian co-ordinates (x, y) and the axis of y runs normal to the straight edge of the plate. Taking the plane of the plate as $z = x + iy$, this entire plane is next conformally mapped by means of a suitable transformation into the interior of a unit circle with center at the origin of a complex plane $\zeta = \rho e^{i\psi}$. It is then possible to satisfy the appropriately transformed conditions of the problem by making use of a Fourier series expansion for $\varphi_1(\rho,\psi)$, this being the potential function into which $\varphi(x, y)$ is transformed. In order to insure rapid convergence of the Fourier series, a functional discontinuity introduced by the character of the applied load is given special treatment.

Numerical values of the coefficients for Fourier series containing from one to six terms are given, and the rapid convergence of the series is indicated. The values of the edge stresses due to the applied load are then calculated. Comparison is made with the stress distributions given by the Boussinesq and Melan solutions. In general, the results of the present study are in line with expectations based on the two previous investigations.

Martin Goland, USA

594. Carl Eckart, "The thermodynamics of irreversible processes—IV. The theory of elasticity and anelasticity," *Phys. Rev.*, Feb. 15, 1948, vol. 73, pp. 373–382.

In the first section of the paper the tensor analysis of displacements, strains, and stresses, including in particular initial stresses, is developed. It agrees with the well-known results of the co-ordinate method. The reviewer would note that the statement of the author that the "theory of elasticity" is based on the wrong assumption that a body relaxes to an unstrained state after removal of the impressed forces is contradicted by the fact that initial strains and stresses have always been considered in the theory.

The second section discusses the derivation of phenomenological laws of metaelasticity (called "anelasticity" by the author). This anelasticity is characterized by the time rate of change of the length in a body element, after relaxation from stresses. Tensor equations are developed in terms of the coefficients of an element of length for the strain tensor. Here again the statement of the author, that the theory of elasticity starts from the false assumption that the relaxed state of a body element does not change with time, is contradicted by the literature of this theory.

A thermodynamic approach follows, based on the equations of continuity, momentum, and energy flow, the latter in terms of heat flow, change of internal energy, and density change. The author then introduces the condition of increase of entropy, but does not show that the system body element or finite body is an isolated system. A relation between the time and space change of the coefficients of the element of length and strains and stresses is derived. It leads to a generalized Stokes' stress tensor of hydrostatic, elastic, and viscous components.

The last section deals with the propagation of weak waves in an anelastic isotropic medium with neglect of thermal effects. The wave equations of dilatation and distortion are formulated. The author suggests the determination of the arbitrary constants in these equations by the observation of the spectra of "relaxation times" (zeros of propagation velocity).

H. J. Reissner, USA

595. M. Z. Narodezky, "Stresses in a nonhomogeneous circular cylinder" (in Russian), *Notes Acad. Sci. USSR (Doklady, Ak. Nauk SSSR)*, Dec. 1, 1947, vol. 58, pp. 1305–1308.

The author derives a solution for a plane state of stress in a circular cylinder made of two different elastic isotropic media, one of which occupies the core and the other the rim of the cylinder.

By making use of the complex variable the author reduces the problem to that of finding two functions of the complex variables for each of the two above media [see for example Timoshenko's "Theory of Elasticity," McGraw-Hill Book Co., Inc., 1934, p. 162]. These functions are then subjected to the boundary conditions, which express: (1) The continuity of stress and deformation on the circle separating the two media; and (2) the equilibrium of forces on the outer circle.

The above expressions are further transformed by means of the Cauchy integral theorem, and they are expanded in series according to the increasing powers of a parameter depending on the elastic constants of the two media. By a suitable choice of constants a rapidly convergent series can be obtained.

The solution is applied to a disk subjected to two diametrically opposite forces, and made of two materials whose moduli are in the ratio of 3 to 4, with the radius of the core 0.9 of the outer radius. The computation shows that under these conditions the difference in stress between a homogeneous and heterogeneous cylinder is not very great. Daniel Rosenthal, USA

596. M. Picone, "Existence and calculation of the solution of a certain problem on the boundary for the equations of elasticity (Esistenza e calcolo della soluzione di un certo problema al contorno per il sistema di equazioni dell'elasticita)," *R. C. Accad. Lincei*, Nov.-Dec. 1947, vol. 3, pp. 427–435.

The direct integration of the differential equations of equilibrium is suggested as a method for solving problems in elasticity. From the equilibrium equations and Hooke's law, expressions for the stresses and displacements in terms of the body forces and rotations are derived.

If the body forces are the derivatives of a potential function of the form $L(x) + M(y) + N(z)$, it is found that the components of the rotation and the tangential stresses are arbitrary constants throughout, and general expressions for the normal stresses and displacement components are given. If the body forces are

polynomials in the co-ordinates, so are the displacement components and the general solution is found by expressing the rotation components and tangential stresses as polynomials satisfying the compatibility equations.

Finally, the case of a trinormal region submitted to an arbitrary system of external body forces is analyzed and it is found that the components of the strain can be obtained by solving three problems of Dirichlet in space and five in the plane.

Ernesto Saleme, Venezuela

597. **S. G. Michlin, "Fundamental solutions of the dynamic equations of the theory of elasticity for nonhomogeneous media"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, July-Aug. 1947, vol. 11, pp. 423–432.

The Volterra vectors of elastic displacements are extended to the case of nonhomogeneous media. These vectors, giving the fundamental solutions for the case of nonhomogeneous media, vanish on the surface of a conoid of longitudinal as well as of transverse waves.

With the aid of these solutions there can be obtained a formula which extends the Stokes formula on this case. If the initial displacements and velocities in a disturbed domain of space are given, the generalized Stokes formula leads to an integro-differential equation for displacements. Z. Bazant, Czechoslovakia

598. **Chih-Bing Ling, "On the stresses in a notched plate under tension,"** *J. Math. Phys.*, Jan. 1948, vol. 26, pp. 284–289.

A solution is developed for the stresses in a semi-infinite plate with a circular notch. Solutions are obtained for a notch varying from a full circular shape to zero and extended to become a mound of full circular shape. As noted by the author, various parts of this problem have been treated by several previous writers.

Using bipolar co-ordinates, the Airy stress function is assumed in the form of a Fourier integral in one of the co-ordinates. The constants of integration are determined by the Fourier integral theorem. The infinite integrals obtained for these constants are evaluated by Cauchy's theorem of residues. The infinite integral contained in the final stress formula must be evaluated numerically. Stanley U. Benscoter, USA

599. **Francis Birch, "Finite elastic strain of cubic crystals,"** *Phys. Rev.*, June 1, 1947, vol. 71, pp. 809–824.

The author uses Murnaghan's theory of finite deformation for cases in which the material is of cubic symmetry. The total (isothermal) strain consists of a finite hydrostatic strain with a superposed infinitesimal strain of general type. The free energy is found in an expression which includes terms of the third order in the strain components, thereby introducing six additional elastic constants, functions only of temperature, for the most symmetrical class of cubic symmetry. The theory is developed from both the Lagrangian and Eulerian points of view.

The effects of pressure on the effective elastic constants are determined, with particular reference to the pressure-volume and pressure-compressibility relations. Comparisons are made with results derived from the power-law potential $(b/r^n - a/r^m)$, and a large measure of accord is found in comparisons with much experimental data of Bridgman. D. N. Allen, England

600. **G. H. Livens and Rosa M. Morris, "The boundary-value problems of plane stress—I,"** *Phil. Mag.*, Mar. 1947, vol. 38, ser. 7, pp. 153–179.

This is primarily an expository paper concerned with a systematic treatment of plane stress boundary-value problems of elasticity by methods of the theory of functions of a complex variable. It correlates the work of several British investigators (W. G. Bickley, A. E. Green, S. Wolgate, R. C. J. Rowland, R. C. Knight, A. C. Stevenson, and G. I. Taylor) on stress distribution in isotropic and aeolotropic elastic plates containing holes.

Although the authors cite some earlier work by M. Kolossoff and N. Muschelišvili, and refer to a short paper by S. Lechnitzky [*C. R. Acad. Sci. URSS*, 1936, vol. 3, p. 3], a large amount of related work by Russian investigators escaped their attention. The reviewer believes that the bibliography contained in this paper would be more representative of the present status of two-dimensional problems of anisotropic elasticity if the following references were included: S. G. Lechnitzky [*Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, 1936, vol. 3, no. 1; 1937 (new series), vol. 1, no. 1; 1938, vol. 2, p. 181; 1941, vol. 5, p. 71]; D. Sherman [*Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, 1945, vol. 9, p. 347]; M. V. Morkovin [*Quart. appl. Math.*, 1943, vol. 1, p. 116; 1945, vol. 2, p. 350]. I. S. Sokolnikoff, USA

601. **K. H. Swainger, "Large displacements with small strains in loaded structures,"** *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 45–48.

The paper considers the problem of a flexible structure in which the displacements are large although the strains are small. The theory is very clearly and concisely developed, and the hypotheses underlying it are stated and discussed in some detail. In the method suggested the displacements are assumed to be made up of two parts, the first of which is guessed so as to closely approximate the final shape of the structure, and the second of which is a small correction needed to insure that the compatibility conditions are satisfied. The resulting procedure differs little from that which would be used if the displacements were small, the difference arising mainly in the boundary conditions to be satisfied.

A simple example is given in detail, and involves the generation of a circular cylinder from a flat plate. The "guessed" part of the displacements represents a right circular cylinder which is generated without stretching the middle surface. Hooke's law is assumed to hold. It is found that a cylinder whose radius is 50 times its thickness may be studied under the assumption of small strains. Bruno A. Boley, USA

Experimental Stress Analysis

(See also Rev. 657)

602. **Max M. Frocht, "Photoelasticity, Volume II,"** John Wiley and Sons, New York, N. Y., 1948. Cloth, 6 × 9¼ in., 505 pp., 438 figs., $10.

The importance of this book to the practicing photoelastician cannot be overemphasized. For him, there is a detailed introduction to the two-dimensional theory of elasticity and many comparisons of both elastic and optical theory and other methods of stress analysis with photoelastic results. Complete information is given for a method of numerical solution of Laplace's equation. In photoelastic applications this permits the sum of the principal stresses to be found directly from the stress pattern.

The research worker will be more interested in the sections which carefully describe experimental techniques and equipment for three-dimensional analysis. As stated by the author, although these techniques are relatively simple to apply it took considerable time and much labor to discover and test them. This time and expense can now be saved and the quality of work can be materially improved. Much of the information presented

is new. Among the material included is: a description of how to eliminate edge effect in the freezing technique when photoelastic Bakelite BT-61-893 is used; a discussion of Fosterite, a new Westinghouse styrene-alkyd resin which is particularly suitable for frozen patterns; and a photoelastic analysis of a thick slotted bar in tension to determine the stress in the thickness direction.

One of the minor criticisms which may be made is that not enough is said about the limitations of present theory, in particular what happens when the rotations of the axes of principal stress do become appreciable with respect to the relative phase retardation. Another minor criticism is that the symbolism in the application of oblique incidence to plane stress may be confusing.

D. C. Drucker, USA

603. Hans Ekstein and Stanley Siegel, "Limits of precision in the determination of lattice parameters and stresses by the Debye-Scherrer method," *Nat. adv. Comm. Aero. Tech. Note No. 1375*, Oct. 1947, pp. 1–22.

The limits of precision in the determination of lattice parameters and stresses by the Debye-Scherrer method of x-ray diffraction are examined to determine their relation to the technique of measurement. The shape and breadth of a spectral line and those of the resulting Debye-Scherrer line are compared theoretically and experimentally after the curves have been matched by adjusting one point on the intensity scale.

It is concluded that spectral-line width (in connection with film grain) is the limiting factor for an increase in accuracy, inasmuch as the geometrical errors have already been reduced greatly. A relative error (error in lattice spacing, divided by lattice spacing) of the order of magnitude of 2×10^{-5} is consequently found to be imposed as an experimentally determined uncertainty in the position of maximum intensity, equal to $^1/_{10}$ to $^1/_{30}$ of the line width. This spacing error corresponds to stress errors of 4380 and 810 psi for steel and zinc respectively. The effect of inhomogeneity of the sample is also discussed.

A. R. Bobrowsky, USA

604. Robert D. Ross, "An electrical computer for the solution of shear-lag and bolted-joint problems," *Nat. adv. Comm. Aero. Tech. Note No. 1281*, May 1947, pp. 1–31.

This paper describes refinements in the use of the electrical ladder-type resistance network analogy for the solution of shear-lag problems, as originally described by R. E. Newton [*Proc. Soc. exp. Stress Anal.*, 1945, vol. 2, no. 2, p. 71]. The refinements are as follows: (*a*) Use of variable resistors so that the values may be changed readily to correspond with the various problems being studied; (*b*) use of electronically stabilized constant-current sources, which eliminates the trial-and-error method of adjusting the input currents.

Wm. H. Sparing, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 582, 585, 611, 617, 626, 628, 631, 632, 633, 634, 636, 638, 645, 646)

605. S. E. Mikeladze, "Bending of a beam on elastic foundation, loaded longitudinally and laterally" (in Russian), *Notes Acad. Sci. USSR (Doklady Ak. Nauk SSSR)*, Jan. 21, 1948, vol. 59, no. 3, pp. 451–454.

A novel form of a Fourier expansion is used for finding the deflection of a beam on an elastic foundation, loaded longitudinally and laterally. Its convergence is too slow for the determination of moments and shears, so a new fast convergent series is written down, by the same formula, for the difference between the deflections of the given beam and the deflections of a similar beam devoid of the axial load and elastic foundation. Since the solution of the latter beam is known, the given problem is also solved.

The presentation suffers from a scarcity of hints regarding the derivation and the significance of the method. No numerical example is given.

A. Hrennikoff, Canada

EDITOR'S NOTE: Dr. Hrennikoff has made an English translation of this paper, which the editor will be glad to loan to interested readers.

606. Pai C. Hu and Charles Libove, "A relaxation procedure for the stress analysis of a continuous beam column elastically restrained against deflection and rotation at the supports," *Nat. adv. Comm. Aero. Tech. Note No. 1150*, Oct. 1946 (publ. in 1947), pp. 1–74.

A relaxation procedure is given for the stress analysis of a continuous beam column on elastic supports, in which it is assumed that each support can be replaced by two independent elastic springs, one deflectional and the other rotational. The procedure employs force and moment distribution to determine (to within the effects of the final residual moments and forces) the deflections and rotations of the supports.

This work is facilitated by the use of special tables, which have been prepared for the case of spans with uniform flexural rigidity, and subject to constant axial compression. The shear forces and moments at the supports, the deflection curve, and the bending moment and shear force diagrams are then obtained successively from simple equations and convenient graphs which have been prepared. An illustrative example is worked out in detail.

H. J. Greenberg, USA

Plates, Disks, Shells, Membranes

(See also Revs. 593, 601, 612, 613, 614, 615, 620, 623, 626, 627, 628, 630, 639, 649, 653, 726)

607. W. A. Nash, "Effect of a concentric reinforcing ring on stiffness and strength of a circular plate," *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 25–29.

The author shows how the maximum deflection and maximum stresses may be reduced by reinforcing the circular plate with a concentric ring. Numerical values are given for the reduction, in the case of uniform normal pressure on a fixed-edge plate.

C. J. Bernhardt, Norway

608. Chi-Teh Wang, "Nonlinear large-deflection boundary-value problems of rectangular plates," *Nat. adv. Comm. Aero. Tech. Note No. 1425*, Mar. 1948, pp. 1–113.

The nonlinear differential equations of von Kármán are studied by the use of finite difference equations for the values at net points. The boundary conditions, for clamped or simply supported edges, and for conditions as to the middle-surface strain appropriate to edge support by an elastically yielding stringer, are similarly treated.

Two methods for solving these nonlinear algebraic equations are applied to the example of a uniformly loaded, simply supported, square plate. In the first the deflections are assumed. These permit solution for stress-function values from one set of linear equations. These values, inserted in the remaining set, provide linear equations for deflection values, hence a new set of

deflection values. The cycle can then be repeated. Such a direct process of successive approximation is, however, found to be nonconvergent. Instead of this, therefore, the second cycle is begun with certain means between the initial and final values from the first cycle. Mathematical justification of this method is not given, but it yields rapid convergence. It gives results consistent with existing results from other methods (Levy, Way), and agrees very closely with test results [R. M. Head and E. E. Sechler, "Normal pressure tests on unstiffened flat plates," *Nat. adv. Comm. Aero. Tech. Note No. 943*, 1944] for $pa^4/Eh^4 < 120$ (where p is the pressure, a the side of the square, h the thickness). For higher values of this ratio, the test deflections exceed the calculated ones. The author suggests that the von Kármán equations may not be valid beyond this point.

The example is also solved by a relaxation method. The assumed deflections permit a relaxation calculation for the stress function from the differential equation. This cycle is repeated until the results cease to change significantly. Close agreement with the preceding method is found.

The author concludes that the large-deflection problems of rectangular plates can be solved by his methods with any boundary conditions, to any degree of accuracy required, and that although still difficult the method is simpler than previous methods for the same accuracy.

A rather full review of previous work and a brief description of the relaxation method are included.

J. N. Goodier, USA

609. **V. Z. Vlasov, "Membrane theory of thin shells of revolution"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, July-Aug. 1947, vol. 11, pp. 397–408.

The author makes use of the equations of the membrane theory of shells to solve several problems involving elliptical, spherical, and parabolic domes, subjected to prescribed loads.

It is shown that the equations of momentless or membrane theory are reducible to Cauchy-Riemann equations if the shells are quadric surfaces with positive Gaussian curvature.

I. S. Sokolnikoff, USA

610. **C. J. Thorne, "Square plates fixed at points,"** *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 73–79.

In this paper a method is described for obtaining an approximate solution to the problem of a flat plate subjected to symmetrical loads. The solution combines a function satisfying the differential equation of lateral equilibrium with a number of biharmonic polynomial functions having coefficients determined from the boundary conditions at selected points along the edges. The number of biharmonic polynomial functions used limits the number of points at which boundary conditions can be satisfied.

Numerical results were obtained for six different cases of loading and support at the edges of a square plate, by satisfying boundary conditions at five points on each edge. The results for a plate with clamped edges under uniform load and under central point load are in excellent agreement with those obtained by other authors. Plots are given of the computed deflection and slope at the edges and it is shown that, in addition to satisfying boundary conditions at the five points used in the solution, the boundary conditions are nearly satisfied all along the edge. The author points out that tables useful in applying this method to other problems are presented in another paper of his ["Symmetrically loaded rectangular plates fixed at points," *Univ. Utah engng. Exp. Sta. Bull.*, no. 34.]

Samuel Levy, USA

Buckling Problems

(See also Revs. 605, 606, 608, 634, 662)

611. **Karl Möhler, "Calculated and observed values of permissible load and buckling of simple and composite wooden beams (Tragkraft und Querkraft von einund mehrteiligen Holzdruckstäben nach Rechnung und Versuch),"** *Bauplan. u. Bautech.*, Feb. 1948, vol. 2, pp. 41–47.

The results of buckling tests with simple and composite wooden struts are compared with German design specifications based on Tetmajer's formula for buckling. The effect of various initial eccentricities is investigated and the well-known fact confirmed that the influence of eccentricity increases with decreasing slenderness ratio. An empirical formula for shear in a composite strut under buckling conditions is proposed.

A. M. Freudenthal, USA

612. **George Gerard, "Critical shear stress of plates above the proportional limit,"** *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 7–12.

The secant modulus method previously used by the author for critical compressive stresses has been used in the present paper in determining critical shear stresses for plates in the plastic domain, which are found by multiplying the elastic buckling stress by a material constant G_s/G, where G is the shear modulus and G_s is the shear secant modulus. The method was confirmed by tests with sheets of aluminum alloy 24S-O, subjected to pure shear. Nondimensional critical-shear-stress design curves are presented for use with aluminum alloys.

The author was apparently not familiar with this reviewer's theory of plastic stability [*Inst. Baustat. T. H. Zurich*, no. 21, 1938] from which it may be shown that for the special cases considered by the author the secant modulus method is indeed a rather good approximation. P. P. Bijlaard, Holland

613. **P. P. Bijlaard, "On the plastic stability of thin plates and shells"** (in English), *Proc. kon. Ned. Akad. Wet.*, 1947, vol. 50, no. 7, pp. 765–775.

In the first part of this paper the author compares the recent test results of Kollbrunner [*Inst. Baustat. T. H. Zurich*, no. 17, 1946] on the plastic stability of thin plates with his former theory of plastic buckling. The comparison is made on the basis of the critical buckling stress and the number of waves developed. He finds that the number of waves produced in the tests is exactly as predicted by this theory and that the discrepancies of the buckling stresses are not more than in the elastic domain.

In the latter part of the paper the author applies his theory to the plastic buckling of shells. He develops equations for the case of a cylindrical shell, and, although he has no test data with which to check these results, he finds certain similarities with the case of the flat plates if it is assumed that the entire shell deforms plastically. Evan A. Davis, USA

614. **P. P. Bijlaard, "On the elastic stability of sandwich plates, I—II"** (in English), *Proc. kon. Ned. Akad. Wet.*, 1947, vol. 50, no. 1, pp. 79–87; no. 2, pp. 186–193.

This paper is an extension of the author's previous papers on this subject.

In Part I the author considers the elastic stability of a rectangular sandwich plate simply supported along all four edges and subjected to compressive forces in the plane of the plate in two directions. It is assumed that the plate as a whole buckles antisymmetrically as a simple sine wave. The outer layers of the plate

are assumed to have an infinite shear modulus, but the middle layer is permitted to suffer shear strain. The resulting torsional shears in the center layer are shown to be small and therefore are neglected. The critical compressive forces are calculated and compared with the more approximate results previously obtained by Van der Neut. Application of the method to other boundary conditions is briefly discussed.

In Part II the buckling of a sandwich plate infinitely long in one direction and loaded by shear forces along the two edges is considered. The symmetrical buckling of the problem of Part I is also discussed. Paul F. Chenea, USA

615. Tsun Kuei Wang, "Buckling of transverse stiffened plates under shear," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 269–274.

An analysis is presented of the stability of a stiffened plate acted upon by edge shear. Essentially, it extends to any number of stiffeners Timoshenko's analysis of the shear stability of plates with one and two stiffeners [*Eisenbau*, 1921, vol. 12, p. 147]. Critical stresses are obtained by the energy method. The deflected shape of the plate-stiffener assembly is expressed by a double Fourier series whose coefficients are obtained by minimizing the expression for the critical shear force.

The rigidity of the stiffeners affects both the shape of the wave pattern and the magnitude of the critical shear stress. For small stiffener rigidity the waves spread across the stiffeners, whose flexural strain energy is therefore included in the energy expression for the assembly. At higher rigidities each panel buckles as a rectangular plate between straight stiffeners. It is stated that, with regard to the stiffeners, "the moment of inertia is the only parameter which should be considered. The torsional effect of the stiffeners is comparatively small and thus is neglected here."

Design graphs for three and four stiffeners and for an infinitely long plate, for panel side ratios up to one, are intended to permit the practical determination of required stiffener rigidities to obtain a given critical shear stress, including the case when different materials are used for the plate and for the stiffeners. However, this use of the paper is unfortunately made almost impossible by numerous oversights, such as missing or mistaken cross references and erroneously interchanged captions of graphs and figures. Without access to the original numerical computations this reviewer was not able to rectify these oversights.

George Winter, USA

616. Robert Schönhöfer, "Direct calculation of strut cross sections and their optimum shapes (Unmittelbare Berechnung von Knickquerschnitten und deren wirtschaftliche Gestaltung)," *Bauplan. u. Bautech.*, Oct. 1947, vol. 1, pp. 117–120.

The author attacks the problem stated, by means of approximate formulas for the radius of gyration. From this and a table for safety factor versus slenderness ratio, the dimensions of the cross section can be determined. The method is applied to solid circular and rectangular cross sections, as well as several thin-walled sections. These are listed in the order of their economy of material, together with modifying conditions.

G. H. Handelman, USA

617. R. Lévi, "Note on buckling, particularly of arches (Note sur le flambement et en particulier celui des arcs)," *Ann. Ponts Chauss.*, May-June 1947, vol. 117, pp. 345–366.

This paper is concerned with the stability of the equilibrium of a curved rod which is acted on by transverse loading and terminal thrusts. The equation of neutral equilibrium is derived as the condition that the total energy of strain and of external loading should be unchanged to the second order in a small displacement.

The displacement caused by the transverse load is assumed known, and this displacement and the virtual displacement which accompanies instability are expanded in the orthogonal functions which are solutions of Euler's equation, $EI\,d^2y/dx^2 + Py = \text{constant}$, for the stability of an analogous straight rod. The length of this straight rod is the distance between the supports of the curved rod, and its flexural rigidity, EI, is the same as the (in general variable) flexural rigidity of the curved rod at the corresponding section. The critical thrust of the curved rod is therefore expressed in terms of the critical loads P_1, P_2, ... of the straight rod.

This relation is discussed in general terms, and, in an appendix, some simple numerical examples are given for a rod of constant flexural rigidity. W. R. Dean, England

618. W. A. Hickman and N. F. Dow, "Compressive strength of 24S-T aluminum-alloy flat panels with longitudinal formed hat-section stiffeners having four ratios of stiffener thickness to skin thickness," *Nat. adv. Comm. Aero. Tech. Note No. 1553*, Mar. 1948, pp. 1–39.

Results of compression tests on 24S-T flat panels with formed hat-section stiffeners are presented in tabular and graphical form. All stiffeners were of 0.040-in. thickness and were riveted to skins which ranged in thickness from 0.032 to 0.102 in. Stiffener spacings and dimensions were also varied. Design charts have not been prepared from this data. John E. Goldberg, USA

619. P. E. Sandorff, "Bending rigidity and column strength of thin sections," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, pp. 833–841.

In structural members having deep thin webs and thin wide flanges the stress distribution in the flanges is not uniform as it is assumed to be in the elementary theory of bending. By the application of elasticity theory, the author has calculated an efficiency factor for the flanges of various unsymmetrical sections including I, Z, and channel sections, and simple and multiple web box beams. This efficiency is defined as the ratio of the total force in the flange to the force which would be expected under the assumption that the flange stress is uniform and equal to the stress at the web connection.

Curves for this efficiency factor are presented and their application to stability problems and other design problems is indicated. The calculations are compared with experimental results and show good agreement. J. O. Hancock, USA

620. D. V. Panov, "On the stability of a bimetallic membrane on heating" (in Russian), *Appl. Meth. Mech. (Prikl. Mat. i Mekh)*, Nov. 1947, vol. 11, pp. 603–610.

This paper is concerned with the investigation of the stability of a shallow bimetallic shell used in a commercial thermostat switch. I. S. Sokolnikoff, USA

621. J. Brunner, "Buckling resistance (Knickstabilität)," *Schweiz. Bauztg.*, July 12, 1947, vol. 65, pp. 379–381.

The author presents curves to simplify the practical determination of buckling loads of bars in structures. The end conditions of a beam are characterized by a factor g, which is the ratio of the angular displacement of the beam to the angular displacement of the supporting joint under the action of a unit torque. Curves showing the length of an equivalent beam simply supported as a

function of this factor q are given. It is shown that when $q = 3$, the beam can be considered approximately a built-in beam. An extension to the case where beams are stressed above the elastic limit is briefly mentioned. John L. Maulbetsch, USA

622. **E. Stagni, "A numerical method for the determination of the critical load in straight columns (Un metodo numerico per la determinazione del carico critico nelle strutture lineari),"** *Ann. Mat. pura appl.*, 1945 (published in 1947), ser. 4, vol. 24, pp. 237–256.

In this paper the differential equations for the buckling of columns are derived from energy considerations. Solutions for columns of variable cross section (using Ritz's method and the method of successive approximations) are discussed in considerable detail. A formula for a corrective term is derived by means of Taylor's series. Equations are brought to a form most suitable for numerical integration.

A straightforward numerical integration employing finite differences is proposed and illustrated by an example based on a pin-ended column of uniform cross section. The method proves to be simple, reasonably fast, and sufficiently accurate for practical purposes.

The basic principles of the numerical integration and the solution of problems that could not be handled by ordinary methods were presented in other papers by the author [*G. Gen. civ.*, Feb. 1947, vol. 85, p. 82; June 1947, vol. 85, p. 264]. (See Rev. 259, 260.) J. H. Meier, USA

623. **N. J. Hoff, B. Klein, and B. A. Boley, "Stresses in and general instability of monocoque cylinders with cutouts—VI. Calculation of the buckling load of cylinders with side cutout subjected to pure bending,"** *Nat. adv. Comm. Aero. Tech. Note No. 1436*, Mar. 1948, pp. 1–30.

A strain-energy theory is developed for the calculation of the buckling load of a circular monocoque cylinder that has a side cutout and is subjected to pure bending. The increment of strain energy due to buckling is derived with the aid of a deflection function which was selected to conform with observed deflections of test specimens. Participation of the reinforcing rings in the buckling is taken into account.

In six tests, the calculated buckling loads ranged from 97 to 155 per cent of the experimental values. H. L. Langhaar, USA

Joints and Joining Methods

(See also Rev. 604)

624. **H. W. Russell, L. R. Jackson, H. J. Grover, and W. W. Beaver, "Fatigue strength and related characteristics of aircraft joints. II. Fatigue characteristics of sheet and riveted joints of 0.040-in. 24S-T, 75S-T, and R303-T275 aluminum alloys,"** *Nat. adv. Comm. Aero. Tech. Note No.* 1485, Feb. 1948, pp. 1–97.

This paper presents the results of a series of direct-stress fatigue tests of aluminum-alloy sheet materials and simple riveted joints. Alclad and bare sheets (0.040 in. thick) of 24S-T, 75S-T, and R303-T275 were used in the program. The effects of coating, notching (scratching), and drilling holes in sheet material were investigated, and several types of riveted joints were tested. Tests of cumulative damage and the effects of 375 F temperature were included.

It was found that notched and unnotched bare sheets had greater long-life fatigue strengths than the corresponding Alclad materials. For short lifetimes, the difference was less. Fatigue strengths were not consistent with ultimate static strengths. Shallow scratches had little effect, but those penetrating the base metal were detrimental. Results at 375 F were about the same as those at room temperature.

Riveted-joint test results indicated that the long-life fatigue strengths of single-row lap joints were in the same relative order as the notch-fatigue strengths. Increasing the number of rivet rows decreased the fatigue strength per rivet. Stiffened lap and butt joints had a considerably higher ratio of long-life fatigue to static strength than simple lap joints.

Harry A. Williams, USA

625. **H. J. Grover and L. R. Jackson, "Fatigue tests on some spot-welded joints in aluminum-alloy sheet materials,"** *Welding Res. Supplement*, Apr. 1947, vol. 12, pp. 215–232.

The paper presents some results of fatigue tests on aluminum sheets, carried out at Battelle Memorial Institute, Columbus, Ohio, and originally published in a series of detailed Nat. adv. Comm. Aero. Advance Restricted Reports. The investigations were carried out on: (*a*) spot-welded stiffened panels; (*b*) sheets with spot-welded attachments; and (*c*) spot-welded lap joints. The conclusions drawn in the paper afford strength values which can be used in aircraft engineering design.

S. K. Ghaswala, India

Structures

(See also Revs. 593, 601, 604, 606, 618, 619, 621, 623, 668, 670)

626. **J. Hadji-Argyris and P. C. Dunne, "The general theory of cylindrical and conical tubes under torsion and bending loads,"** *J. roy. aero. Soc.*, Feb. 1947, vol. 51, pp. 199–269; Sept. 1947, vol. 51, pp. 757–784; Nov. 1947, vol. 51, pp. 884–930.

An extensive analysis is made of the stresses due to torsional and bending loads in single or many-celled cylindrical or slightly tapered tubes (wings) of arbitrary cross section, formed of thin sheet with longitudinal booms. It is assumed that the cross-sectional form is maintained by a closely spaced system of diaphragms, which are rigid to deformation in their planes (the tube cross sections) but permit normal (warping) displacements. Differential equations are obtained in general form covering the effects variously described as axial constraint, diffusion, and shear lag and nonuniform torsion. Sheet buckling is allowed for by assuming a reduction of sheet thickness. A Bredt-Batho formula relating twist and shear stress is obtained without restricting the loading to torsion. As a consequence of the rigid-diaphragm assumptions, the warping depends only on the shear stress round the section, and the shear stress at a fully axially constrained section is statically determinate for any loading.

To meet conditions at a root with prescribed fixity, and at a section at which concentrated loads act, an investigation is made of the existence and form of a set of self-equilibrating end loads which give stresses proportional to the product of a function of the axial co-ordinate and a function of the arc co-ordinate in the section. In an n-boom structure with sheet effective only in shear, there are $n - 3$ such "eigenloads." They take into account the discontinuities introduced by booms and wall junctions of a multicell tube. The differential equations for these eigenfunctions have coefficients depending on the variation of sheet thickness. Variations which permit solution in polynomial, hyperbolic, and Bessel functions are treated.

The method is applied, in the first installment, to concentrated torque loading at any section, and generalized to an arbitrary spanwise torque distribution. Curves are given for

several cases with and without taper. The multicell 4-boom tube is reduced to a single-cell tube equivalent with respect to the eigenloads.

In the last two installments the authors continue with a unification of the theories of bending and torsion. The practice of calculating axial constraint stresses on the basis of torque about the flexural axis is found to be incorrect. Corrections for the whole series of papers are given on page 928 of the last installment. J. N. Goodier, USA

627. Wilhelm Nerlich, "Design and reinforcement of circular concrete foundation slabs [Berechnung und Bewehrung von Kreisplatten (Stahlbeton) als Gründungsplatten]," *Bauplan. u. Bautech.*, Jan. 1948, vol. 2, pp. 5–9.

Formulas for the bending and twisting moments and shear forces in a circular plate loaded symmetrically and unsymmetrically are collected for use in designing reinforced concrete foundation slabs and footings. Design curves are presented and layout of the reinforcing steel is discussed.

Paul F. Chenea, USA

628. N. M. Newmark, "Design of I-beam bridges," *Proc. Amer. Soc. civ. Engrs.*, Mar. 1948, vol. 74, pp. 305–330.

Recommendations are presented for the design of the slab and beams of I-beam highway bridges. Consideration is given to simple and continuous spans, to right and skew bridges, and to bridges in which T-beam action is obtained by the use of either shear connectors or concrete beams. The treatment is based on both analyses and tests. The practical significance of the design recommendations is discussed in the light of current practice and current specifications. Enrico Volterra, Italy

629. P. Cicala, "Effects of cutouts in semimonocoque structures," *J. aero. Sci.*, Mar. 1948, vol. 15, pp. 171–179.

This investigation extends the study of stress distributions in a semimonocoque structure with a cutout and with frames rigid in their own plane, to the case in which the frames are flexible. An analysis is made of a thin-skin circular cylinder reinforced by equally spaced stringers and circular frames.

A solution is developed for the "corrective stresses" representing the perturbations arising from the cutout. This considers the deflection of the frames figured according to the theory of curved beams. It is shown that the peak values of stress near the cutout are many times greater than those given by rigid-frame theories.

Evaluations are made, and shear flows are indicated by means of curves for torsion of the circular cylinder with various angles of cutout opening. These results compare reasonably with test results obtained by Langhaar and Smith.

M. V. Barton, USA

630. F. E. Richart, "Laboratory research on concrete bridge floors," *Proc. Amer. Soc. civ. Engrs.*, Mar. 1948, vol. 74, pp. 288–304.

A program of systematic research was begun in September 1936 with the purpose of making investigations and tests of reinforced concrete slabs to determine their behavior under varying conditions, and to develop information which will advance the art of bridge building.

A combination of analytical and experimental study has been employed throughout the work with the main object of covering three definite phases: (1) Mathematical analysis along conventional lines; (2) experimental research on structures, with results analyzed and compared with theoretical findings; (3) the development of simplified design procedures for rapid and easy use. Enrico Volterra, Italy

631. J. Griveaud, "Studies and propositions concerning Vierendeel girders (Études et propositions relatives aux poutres échelles)," *Ann. Ponts Chauss.*, vol. 117, May-June 1947, pp. 311–344.

A general theory for the analysis of Vierendeel bridge girders is given. The author considers the computation practicable only when particular forms for the upright members are assumed.

A detailed analysis is given for the case of a uniform Vierendeel girder with rectangular bays. In this case the author might more simply have used finite difference equations, as given for example by Bleich and Melan in their book, "Die Differenzengleichungen und ihre Anwendung in der Baustatik," Springer, Berlin, 1927. It would seem that the relaxation or Hardy-Cross method is more appropriate for the general case.

P. C. Dunne, England

632. C. P. Siess, "Composite construction for I-beam bridges," *Proc. Amer. Soc. civ. Engrs.*, Mar. 1948, vol. 74, pp. 331–353.

The results of analytical and experimental studies of composite construction for I-beam highway bridges which are presented in the paper may be divided into three groups: (1) Comparative design studies of I-beam bridges, to determine the savings in weight possible from the use of various types of composite beams; (2) studies of the behavior of composite beams; (3) studies of the behavior of shear connectors for use in composite I-beam bridges. Enrico Volterra, Italy

633. Palamede Borsari and Ai-ting Yu, "Shear lag in a plywood sheet-stringer combination used for the chord member of a box beam," *Nat. adv. Comm. Aero. Tech. Note No. 1443*, Mar. 1948, pp. 1–57.

A theoretical analysis is presented of shear lag in the plywood used in the sheet-stringer chord member of a box beam of constant section. Face grain of the plywood is at 45 deg to the longitudinal axis and the elastic properties of the plywood in this orientation are considered. Assuming a parabolic variation of longitudinal strain at any transverse section, and using the principle of minimum energy, expressions for displacement and longitudinal and shearing strains are obtained. Strain measurements made on the prototype beam show good agreement with the analytical results. John E. Goldberg, USA

634. Paul Kuhn and James P. Peterson, "Strength analysis of stiffened beam webs," *Nat. adv. Comm. Aero. Tech. Note No. 1364*, July 1947, pp. 1–60.

A semiempirical method is developed for stressing thin-webbed shear beams. The method is basically a modification of the senior author's previous report [*Nat. adv. Comm. Aero. Rep., No. 697*, 1940].

Formulas and charts are presented for the critical shear stress, the average and maximum stresses in the stiffeners, the angle of diagonal tension, the allowable stresses in stiffeners, web design, rivet design, bending moments in flanges, and shear stiffness of web. These are compared with experimental results on 122 beams. The agreement between the formulas and tests is considered satisfactory; apparently no measurements were made of

the additional compressive stresses in the beam flanges. A table covering 34 test beams shows that 29 beams failed in the predicted way with the ratio of ultimate load to predicted ultimate load ranging from 0.83 to 1.34. The agreement with the tests of Lahde and Wagner and with Levy's theory [*Nat. adv. Comm. Aero. Tech. Note Nos. 962, 1009*] for loads small compared to the critical is fair, though the present method yields somewhat less conservative results.

Similarly, a comparison made by this reviewer with his theoretical results obtained in the Netherlands during the war [*Nat. LuchtLab. Amsterdam Versl.*, S295] indicates that this theory for infinitely long plates is somewhat more conservative than the present semiempirical method. W. T. Koiter, Holland

625. **Filippo Arredi, "Thermal stresses in arch dams (Le sollecitazioni indotte dalle variazioni de temperatura nelle dighe ad arco),"** *G. Gen. civ.*, Feb. 1947, vol. 85, pp. 55–70.

In this article expressions for thermal stresses in thick-walled circular cylinders are derived and used for the case where the temperature of one surface is a sinusoidal function of the time. Considering dams as sections of cylinders the author finds values for thermal stresses, which he compares to empirical data used in Italy for dam computations. Conditions are assumed to be uniform along the height of the dam. The only edge effect considered is that of the supports on the sides.

John L. Maulbetsch, USA

636. **Letitia Chitty, "On the cantilever composed of a number of parallel beams interconnected by crossbars,"** *Phil. Mag.*, Oct. 1947, vol. 38, pp. 685–699.

The problem considered arises, for example, in the study of tall buildings under wind load, where the crossbars represent the floor girders. The external load in the plane of the structure is assumed uniform. The crossbars are assumed to be of constant bending stiffness and their shortening is neglected. They are further replaced by an equivalent continuous medium.

The following cases are dealt with: (1) Two equal beams of constant section; (2) two unequal beams of constant section; (3) any number of unequal beams of constant section, the axial strain in the beams being neglected; (4) two unequal beams, with the structure divisible into zones of constant stiffness of beams and crossbars; (5) two beams with continuously varying sections and constant stiffness of crossbars.

H. F. Michielsen, Holland

637. **Hans Leitner, "Reinforcement of structures against wind and earthquake forces (Über Verstärkung von Gebäuden gegen Wind- und Erdbebenkräfte),"** *Bauplan. u. Bautech.*, Jan. 1948, vol. 2, pp. 11–14.

The author discusses the influence of the relative rigidity of columns and girders on the distribution of bending moments due to horizontal forces in the columns of one-bay, multiple-story rigid frames. The rather obvious conclusions are reached that rigid columns tend to a condition approaching that of two cantilevers, while rigid girders reduce the column moments by enforcing points of inflections between stories.

A. M. Freudenthal, USA

638. **G. Zender and C. Libove, "Stress and distortion measurements in a 45-deg swept box beam subjected to bending and to torsion,"** *Nat. adv. Comm. Aero. Tech. Note No. 1525*, Mar. 1948, pp. 1–36.

This gives stress and deflection measurements for an untapered 248-T box beam, representing the primary structure of a full-span, two-spar, single-cell, 45-deg swept wing with a carry-through bay. The specimen was subjected to symmetrical tip bending and twisting loads with stresses below the proportional limit. The authors state that the important effect of sweepback in the case of bending was a considerable build-up of normal and vertical shear stresses in the rear spar near the fuselage attachment. This was not noticed in the torsion case.

Bending and torsion stresses in the outer portions seem to agree with values given by the elementary formulas. It is shown that spar deflections may be estimated by analyzing the outer portion as an ordinary cantilever and adjusting for the flexibility of the inboard portion. John E. Goldberg, USA

639. **Robert S. Levy, "Effect of bending rigidity of stringers upon stress distribution in reinforced monocoque cylinder under concentrated transverse loads,"** *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 30–36.

Using the least-work principle, a theoretical investigation is carried out to determine the effect of bending resistant stringers located at the points of application of concentrated transverse loads, and the resulting shear stresses are compared with those obtained by neglecting the bending rigidity of the stringers.

Calculations for a numerical example, with applied loads diametrically opposed, indicate that neglect of the bending rigidity of stringers results in calculated maximum shear stresses approximately 20 per cent on the conservative side in the fields of load application, and 50 per cent on the unsafe side in intermediate fields. To verify the results, an experiment with strain-gage rosettes is reported, which indicates good agreement between the shear stresses measured and those calculated by the present method. R. G. Boiten, Holland

Plastic Flow, Failure

(See also Revs. 612, 613, 654, 666, 667)

640. **L. Sokolov, "Experimental investigation of the temperature and speed dependance of the resistance to plastic deformation in metals and amorphous bodies"** (in Russian), *J. tech. Phys. (Zh. tekh. Fiz.)*, 1947, vol. 17, no. 5, pp. 543–548.

Specimens 10 mm in diameter and 20 mm high made of lead, tin, zinc, aluminum, copper, and nickel, and specimens 200 mm in diameter and 100 mm high made of tar, plasticine, and glass were submitted to static compression with various rates of deformation from 0.01 to 1 mm per sec, and dynamic compression by means of a Charpy pendulum developing a rate of deformation of about 2000 mm per sec.

The load-deformation diagram in static tests and the work-deformation diagram in dynamic tests were plotted, and empirical relations between true stress and speed of deformation were derived for a constant value of 20 per cent of the permanent set at various temperatures. Daniel Rosenthal, USA

641. **M. G. Winterstein, H. J. McDonald, and J. T. Waber, "Determination of physical chemical factors in stress-corrosion cracking of mild steel,"** *Welding Res. Supplement*, Dec. 1947, vol. 12, pp. 723–726.

The rate and direction of growth of a crack are found to depend upon the localized electrochemical conditions induced by mechanical forces. The correlation found between cracking time, extent of aging, and "free" nitrogen indicates that the steel can be made relatively more resistant, but not completely resistant,

to stress-corrosion cracking. The removal and subsequent reintroduction of nitrogen eliminates and then restores the cracking tendency in mild steel.

A new type of test procedure, employing a wire specimen under constant load, is described, which gives intercrystalline cracking consistent with normal stress-corrosion cracking. In a single series of tests with the new apparatus, the energy of activation of the stress corrosion of mild steel is determined, using a standard mixed nitrate solution.

The results indicate that: (1) The reaction involved in stress-corrosion cracking is probably of the diffusion type, since the activation energy is too low for the usual direct chemical attack; (2) since the energies of activation found for nitrate solution and for buffered acetate solution are similar, it appears that the type of solution used is secondary to the chemical composition of the steel. Apparently, the type of ions present does not completely govern the rate of the stress-corrosion reaction.

Clayton O. Dohrenwend, USA

642. H. W. Swift, "Length changes in metals under torsional overstrain," *Engineering*, Apr. 4, 1947, vol. 163, pp. 253–257.

The author reports torsion tests on hollow and solid specimens of brass, stainless steel, aluminum, cupronickel, copper, and low and medium carbon steels, in which axial elongation as well as the angle of twist are measured, for large plastic deformations. A specially constructed gage and a torsional loading system free of axial constraint were developed for this work. Test results showed that axial extension in general accompanied torsional strain in work-hardened materials, while no noticeable axial extension was found in annealed material. The extension was greatest in brass and austenitic steels, least in carbon steels. A reversal in loading following a high degree of twisting produced a transient axial contraction in the bar.

To explain this phenomenon the author assumes that slip and rotation accompanying the deformation give rise to preferential strain-hardening directions and hence to an anisotropic behavior of the material. Face-centered cubic structures are more susceptible to this effect than body-centered crystals, because of their limited direction of slip. Louis F. Coffin, Jr., USA

643. T'ing-Sui Kê, "Experimental evidence of the viscous behavior of grain boundaries in metals," *Phys. Rev.*, Apr. 15, 1947, vol. 71, pp. 533–546.

The mechanical behavior of grain boundaries in metals has been a subject of constant controversy. The present research is designed to examine thoroughly the mechanical behavior of grain boundaries in metals in a quantitative manner.

A simple torsional apparatus has been devised for measuring four types of anelastic effects at very low stress levels, namely: internal friction at low frequencies; variation of dynamic rigidity with temperature; creep under constant stress; and stress relaxation at constant strain. All four types of anelastic effects have been studied in 99.991 per cent polycrystalline aluminum as well as in single crystal aluminum; these effects are practically absent in single crystal aluminum.

The four types of anelastic effects observed in polycrystalline aluminum are completely recoverable and are linear with respect to the applied stress and prior strain. They satisfy within experimental error the interrelations derived by Zener from Boltzmann's superposition principle. These are consistent with the viewpoint that the grain boundaries behave in a viscous manner.

The maximum amount of shear stress relaxation in polycrystalline aluminum determined by the four types of anelastic measurements is about 33 per cent. This is in good agreement with the theoretical value of 36 per cent calculated by assuming the grain boundary to be viscous. The heat of activation associated with the viscous slip along the grain boundaries has been found to be 34,500 cal per mole. The coefficient of viscosity of the grain boundaries in aluminum, estimated by using this heat of activation, is consistent with that of molten aluminum at the same temperature. Similar anelastic effects have also been observed in polycrystalline magnesium, indicating that the viscous behavior is common to all metals. Frederick Seitz, USA

644. K. H. Swainger, "A new criterion of yielding in metals," *Nature, Lond.*, May 31, 1947, vol. 159, pp. 741–742.

The author proposes as a criterion for yielding of metals (and tentatively also for rupture) the relation $A = S + S_o T/[\sqrt{2}(1 + q)T_o]$. Here **A** is the limiting traction (stress-vector) at which the material fails, and has the tensile and shear components **S** and **T**, so that $A^2 = S^2 + T^2$; S_o is the simple tensile failure stress, T_o the simple shear failure stress, and q is Poisson's ratio. For simple shear failure S vanishes and $A = T = T_o$, and the above formula becomes $T_o/S_o = 1/[\sqrt{2}(1 + q)]$, which corresponds to experimental results.

The reader is referred to an as yet unpublished paper for the reasons for this formula. This reviewer questions whether failure is governed by a limiting traction.

M. Reiner, Palestine

645. M. I. Rosovski, "Bending of an actual heated cantilever with nonuniform temperature distribution" (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), 1947, vol. 17, no. 6, pp. 657–660.

An analysis is given of a cantilever beam of material not following Hooke's law, acted upon by a periodic force at the free end. The built-in end is held at constant temperature, and a stationary temperature regime is assumed. It is further assumed that the thermal strains are proportional to the temperature, and that the generalized elastic constants do not depend on temperature. Equations are derived for the deflection at the free end. It is shown that, as the frequency of the pulsating force increases, the behavior of the member approaches that of ideal elasticity.

George Winter, USA

646. Dimitry Morkovin and Omar Sidebottom, "The effect of nonuniform distribution of stress on the yield strength of steel," *Univ. Ill. engng. Exp. Sta. Bull.*, Dec. 18, 1947, no. 372, pp. 3–72.

A study was made to determine whether the stress at which yielding starts in a steel member subjected to nonuniformly distributed stress is larger than the static tensile yield point of the material. A number of investigators have reported that first yielding occurred in a beam, I-bar, etc., at stresses considerably higher than the yield point of the material as determined from a test specimen in which the stress on any section was approximately uniform.

The authors attribute these inconsistencies to erroneous interpretation of data caused by the difficulty of determining the load at which yielding started, or to the varying stress level at which an upper yield point may be exhibited under different stress conditions by some steels.

Theoretical moment-strain diagrams for beams of various cross sections are presented. Methods (developed by Herbert, Upton, and Nadai) of interpreting experimental moment-strain (at the extreme fiber) diagrams to obtain stress-strain relationships for the material are utilized in analyzing actual test data

on beams. Test data are also presented for tension members with abrupt changes of section. Differences in the initiation and spread of plastic zones in a tension member from that in a beam are discussed, to show the reason for discrepancies reported by various investigators, and to illustrate the changes caused by varying levels of upper yield point.

The experimental and analytical results indicate that the stress required to start yielding in the presence of a stress gradient is *not* higher than that required in a uniform stress field.

T. J. Dolan, USA

647. P. P. Bijlaard, "On the restricted applicability of the principle of least work in the plastic domain" (in English), *Proc. kon. Ned. Akad. Wet.*, 1947, vol. 50, no. 4, pp. 397–405.

This paper deals with the variations of the elastic and plastic strain energies of a yielded strip of a steel plate. The background of the work presented is contained in several previous papers by the same author and many references are made to this former work.

Emphasis is placed upon the transitional domain between the elastic and the plastic region. During this transition the state of stress must change from one corresponding to minimum elastic strain energy to one corresponding to minimum shear strain energy. Proof is given that the total strain energy, i.e., elastic plus plastic, does not in general acquire extreme values in this transitional domain and hence each component must be considered independently. Different conditions under which the strain energy can be varied and the restricted use of the results of the variations in terms of possible applications are discussed.

Evan A. Davis, USA

648. B. Gross, "On creep and relaxation—II," *J. appl. Phys.*, Mar. 1948, vol. 19, pp. 257–264.

A phenomenological theory of the elastic aftereffect is formulated with the aid of the principle of superposition. In a previous paper [*J. appl. Phys.*, 1947, vol. 18, p. 212] the author developed the theory of the transient effects which are caused by the sudden application of a constant load or a constant deformation. In the present paper, the theory of the steady-state behavior under alternating load and deformation is given. Relations are established between the loss factor, the storage factor, the distribution functions, and the Laplace transforms of the creep function and of the relaxation function.

It is pointed out that the theories of the elastic aftereffect, of dielectric absorption, and of linear electric circuits are formally equivalent, only the names having to be changed; load corresponds to voltage, deformation to quantity of electricity; rate of creep and elastic modulus function correspond to *indicial* admittance and steady-state impedance. D. C. Drucker, USA

649. W. P. Kerkhof, "Calculation of pressure vessels in connection with investigation of the occurrence of brittle fractures (Berekeningen van apparaten en hiermede verband houdende beschouwingen over het optreden van brosse breuken)," *Ingenieur, 's Grav.*, Feb. 20, 1948, vol. 60, pp. MK.13–21.

Below a temperature of 350 C failure by fatigue has to be taken into account, and for this the author recommends the von Mises-Hencky criterion, as modified by Roš for the welds.

For higher temperatures, the allowable stress has to be based on the allowable strain, transcrystalline and intercrystalline fracture. If Bailey's theory of creep is applied and if it is assumed that transcrystalline fracture occurs when the strain in any direction surpasses a certain amount, it appears that as far as the strain and transcrystalline fracture is concerned the same modified von Mises-Hencky criterion may be applied. For intercrystalline fracture the author assumes the "amount of strain energy" to be the limiting factor. According to his formulas he means by the amount of strain energy the strain energy due to the change of volume. P. P. Bijlaard, Holland

650. B. M. Rovinski and T. V. Tagoonova, "Plastic deformation and the lattice parameter" (in Russian), *J. tech. Phys. (Zh. tekh. Fiz.)*, Oct. 1947, vol. 27, pp. 1137–1142.

The authors investigated the influence of plastic deformation on the change of lattice parameter of "armco" iron, copper, and two copper alloys; one containing 19.92 atomic per cent Zn, the other 17.4 atomic per cent Al. To eliminate the influence of the residual stress on the lattice parameter, the experiments were performed on powders obtained by filing, rather than on plastically deformed blocks of metal. The influence of cold working was then gradually removed by heating the filings to various temperatures until full recrystallization was achieved. X-ray-diffraction back-reflection diagrams were taken in all cases, but the lines were too diffuse to permit lattice parameter determination for the "as filed" condition and for the heat-treatment at the lowest temperature. However, considerable line broadening still was present for other treatments below the recrystallization temperature. It was therefore concluded that for these treatments the grain still was plastically deformed. Despite this circumstance the measurements failed to reveal change in lattice parameter greater than the probable error (0.01 per cent) due to plastic deformation.

On the basis of these experiments it was concluded that the change of lattice parameter reported by Wood and Burgers was produced by the residual stress and not by plastic deformation. On the other hand, changes reported by Schafer for 50 atomic per cent Fe Al alloy may be due to phase changes (from ordered to disordered state) produced by plastic deformation.

D. Rosenthal, USA

651. U. R. Evans and M. Tchorabdji Simnad, "The mechanism of corrosion fatigue of mild steel," *Proc. roy. Soc. London, Ser. A*, Feb. 11, 1947, vol. 188, pp. 372–392.

This paper attempts to establish the mechanism of corrosion fatigue and to investigate the possibility of its prevention by cathodic currents. Two-stage tests were conducted on mild-steel rods, 0.1 in. diam., subjected to alternating stresses of various intensities. A 0.1-mol potassium-chloride solution was applied during the first stage, and the specimens were subjected to the same stress range in air during the second. In one series, cathodic currents were superimposed. Chemical and electrochemical measurements were made on all tests.

Results indicate that wetting with a corrosive liquid during the first stage for a time longer than a certain critical period greatly reduces total fatigue life, but the application of the liquid for a still longer period may actually cause the total life to increase. Alternating stresses increase both the rate of mechanical damage and the rate of chemical corrosion. Small cathodic currents decrease the fatigue life, but a completely protective current exists at which corrosion fatigue is entirely eliminated.

E. O. Stitz, USA

652. L. Sokolov, "Influence of the degree of deformation on the dependence of stress upon speed" (in Russian), *J. tech. Phys. (Zh. tekh. Fiz.)*, Jan. 1948, vol. 18, pp. 93–97.

Cylindrical specimens of lead, tin, aluminum, zinc, copper, nickel, ten kinds of steel, and brasses with different percentages of

zinc were subjected to compression at speeds of 0.01 to 2000 mm per sec. From these experiments, described in an earlier paper [*J. tech. Phys.*, 1946, no. 4, p. 437], the author concludes that for high-melting metals the speed coefficient decreases with increase of deformation at 20 C, and increases at high temperatures; for low-melting metals the speed coefficient increases at 20 C and higher.

From the diagram for copper at 900 C it is deduced that the relation between the plastic stress and the velocity is most conveniently expressed by a double logarithmic relation, as indicated by Rejto and others. D. R. Mazkevich, USA

653. W. F. Brown, Jr., and George Sachs, "Strength and failure characteristics of thin circular membranes," *Trans. Amer. Soc. mech. Engrs.*, Apr. 1948, vol. 70, pp. 241–251.

An experimental study is presented of the deformation and failure under hydrostatic loading of thin circular membranes, made from annealed electrolytic copper, hard-rolled electrolytic copper, and oxygen-free high-conductivity copper.

Particular emphasis is placed on instability phenomena, the data being compared with a theory for the relationship between stress, strain, and the slope of the stress-strain curve at the maximum point in the load-strain curve. In general such relationships can be found in terms of some equivalent-stress versus equivalent-strain curve, which may be transformed into the stress-strain curve of any conveniently determined stress and strain for a given method of loading. The data presented in the paper indicate that such an equivalent-stress versus equivalent-strain curve exists for copper, and the authors are able to show good correlation between predicted and measured values of instability strain for the materials tested. The inherently high instability strain obtained in these tests suggests the use of such "bulge" tests as a means of obtaining equivalent-stress versus equivalent-strain curves up to high strains for materials having such curves.

The authors' closure in reply to the discussion points out that theoretical methods of predicting the shape of a hydrostatically loaded diaphragm which neglect strain hardening may lead to incorrect results. M. B. Millenson, USA

Design Factors, Meaning of Material Tests

(See also Rev. 646)

654. D. Tabor, "A simple theory of static and dynamic hardness," *Proc. roy. Soc. London, Ser. A*, Feb. 4, 1948, vol. 192, pp. 247–274.

The Hertz theory is applied first to the static indentation hardness test, it being shown that the decrease in curvature of the surface of indentation, which occurs when the indenting load is removed, may be computed from this theory. The computed values agree with measurements by Foss and Bromfield.

The Hertz theory is then applied to the dynamic indentation hardness test, in which an elastic indenter is dropped on the specimen from a given height and the height of rebound and diameter of indentation are measured. Formulas are derived for the average dynamic yield pressure P as a function of the measurable parameters of the test. The formulas check with experimental data for many different materials, and they provide an explanation for the observed difference between static and dynamic hardness numbers.

It is shown that the observed relation between the hardness number (mean pressure on the punch) P_m and the indentation diameter, for materials such as annealed copper and mild steel which are subject to work hardening, can be computed directly from the stress-strain curve $Y(\delta)$ of the material by (1) letting $P_m = cY$, where c has a value of about 3, and (2) by taking the equivalent strain δ in the hardness test as that of the deformed metal at the edge of the indentation. The value $c = 3$ agrees with the theoretical results of Hencky and Ishlinsky, based on the energy of distortion theory of yielding.

The theory gives the correct form for Meyer's empirical relations connecting the load, indentation diameter, and ball diameter in a Brinell test, for a material with a stress-strain curve described by $Y = b\delta^x$ (where b, x are empirical constants). The theory is applied also to give a simple relation between Vickers hardness number and the stress-strain curve of the material.

Walter Ramberg, USA

Material Test Techniques

(See also Revs. 641, 642, 643, 654, 663, 664)

655. Henry W. Foster, "A method of detecting incipient fatigue failure," *Proc. Soc. exp. Stress Anal.*, 1947, vol. 4, no. 2, pp. 25–31.

This is a description of experimental technique whereby small wires, which are connected to electrical circuits, are cemented in critical areas where fatigue cracks are expected to develop. The wire breaks if a crack develops under it, and it is thus possible to detect fatigue cracks before complete failure of the member.

John L. Maulbetsch, USA

Mechanical Properties of Specific Materials

(See also Revs. 624, 625, 640, 641, 642, 643, 651)

656. L. R. Jackson, H. C. Cross, and J. M. Berry, "Tensile, fatigue, and creep properties of forged-aluminum alloys at temperatures up to 800 F," *Nat. adv. Comm. Aero. Tech. Note No. 1469*, Mar. 1948, pp. 1–48.

Tensile strength, fatigue strength, and creep tests at elevated temperatures are reported on XB18S, 18S, 24S, and 32S forged alloys. The authors conclude that although an interrelation between these properties should exist, they were unable to find any simple correlation between any of them.

D. R. Mazkevich, USA

657. A. A. Padmos and J. de Vries, "Stresses in glass and their measurement," *Philips Tech. Rev.*, 1947–1948, vol. 9, no. 9, pp. 277–284.

In the molding or blowing of glassware, stresses due to unequal shrinkage arise during cooling, which may cause the glass to crack. The same is the case when fusing together parts whose coefficients of expansion differ appreciably. To find out whether stresses are present in glass, the authors examine the double refraction induced by them. A beam of polarized light passes through the glass object to be examined, then through a "red plate" and a nicol and finally reaches the eye of the observer. The double refraction of the object gives rise to characteristic color effects and the observer judges the stresses accordingly. The magnitude of the double refraction can be measured by means of a compensator.

To prevent the stresses referred to, however, it must first be determined whether the materials possess those expansion coefficients which make them suitable for fusing together. The Philips works employ a method in which two small square plates of glass or metal are fused together to form a simple plate. The

stresses in this plate, due to unequal shrinkage, are then deduced from its double refraction, and this indicates the ability of the two materials to be fused together. C. Massonnet, Belgium

658. **Paul Baumgarten, "Research on the impregnation of soft limestone (Recherches sur l'imprégnation des pierres en calcaire tendre),"** *Bâtim. Trav. publics*, Oct. 15, 1947, ser. D, pp. 2–11.

Since soft limestone is readily quarried and cut, great savings in building costs can be made if its physical properties can be satisfactorily improved without excessive cost. As a result of previous work on the impregnation of metal castings, it was decided to carry out a relatively complete investigation of impregnating procedure and its effect for soft limestone.

The impregnating materials investigated were sodium silicate, an emulsion of polyvinyl acetate, a copolymer of vinyl acetate and vinyl chloride, urea resin, a commercial phenolic solution, and melamine resin. The quality of impregnating procedure as well as impregnating material was measured by its effect on the compressive strength and modulus of elasticity, scratch hardness, abrasion resistance, deformation at fracture, and resistance to freezing and thawing.

For single impregnations, the sodium silicate and the phenolic solution were found to be the most satisfactory, giving increases of the modulus of elasticity of about 10 to 1 and of compressive strength of 6 to 1. An impregnation of sodium silica followed by the phenolic solution gave even better results.

Frank J. Mehringer, USA

659. **F. Stüssi, "Elements of wood engineering (Über Grundlagen des Ingenieurholzbaues),"** *Schweiz. Bauztg.*, June 14, 1947, vol. 65, pp. 313–318.

Three topics are considered: the strength and elastic properties of wood in compression or tension at an angle to the grain, bolted joints, and spaced compression members. The formulas given for strength in compression or tension at an angle to the grain have previously been proposed [for example, see C. F. Jenkin, "Report on materials of construction used in aircraft," *Aero. Res. Comm. Lond.*, 1920]. H. W. March, USA

660. **R. W. Mebs and D. J. McAdam, Jr., "Elastic properties in tension and shear of high-strength nonferrous metals and stainless steel—effect of previous deformation and heat-treatment,"** *Nat. adv. Comm. Aero. Tech. Note No. 1100*, Mar. 1947, pp. 1–105.

Supplementing their previously published data with data from independent investigations, the authors present a complete résumé of their work on the influence of plastic deformation and of annealing temperature on the tensile and shear elastic properties of nickel, monel, Inconel, copper, 13:2 Cr-Ni steel, 18:8 Cr-Ni steel, and aluminum-monel.

The relationship between hysteresis and creep is first obtained by repeated cyclic stressing of annealed stainless-steel specimens over a constant load range, as an aid in devising a loading-time schedule to be used in measurements of elastic properties. The authors point out that the time schedule is very important in such measurements, and that in the present tests the measurements were made only after holding each load for a period of two minutes, thereby allowing the positive and negative creep to reach low rates so that accurate and sensitive set measurements could be made.

Use is made of stress-strain curves to derive the variation of secant modulus with stress, and of stress-set curves to derive five proof stresses used as indexes of the elastic or yield stress. The authors point out that both the stress-strain and stress-set relations are necessary for a complete representation of the tensile elastic properties of a metal.

The variation of the proof stress and modulus of elasticity with plastic deformation or annealing temperature is explained in terms of the relative dominance of internal stress, lattice-expansion or work hardening, and crystal reorientation.

Considerable attention is also directed to the variation of Poisson's ratio with plastic deformation and annealing temperature. S. S. Manson, USA

661. **L. Locati and R. Di Carlo, "On the damping effect of some copper alloys (Sullo smorzamento di alcune leghe di rame),"** *Metallurgia ital.*, Sept.-Oct. 1947, vol. 39, pp. 201–205.

The logarithmic decrement for several alloys was determined by recording the decay of torsional oscillations in a cylindrical specimen 8 mm in diam and 65 mm in length, carrying a heavy flywheel at the lower, free end and fixed at the upper end. Tests were made of binary alloys of copper-nickel, copper-zinc, and copper-manganese with and without heat-treatment.

The copper-nickel and copper-zinc alloys showed considerable damping, which decreased with decreasing stress; the damping was nearly independent of alloy content and was reduced greatly by annealing. The copper-manganese alloy showed a minimum of very low damping at 15 per cent manganese; for 50 per cent manganese the damping was considerable and was independent of stress. The peculiar behavior of the copper-manganese alloy is attributed to a second phase of crystal structure which is latent at 15 per cent manganese but is present at 50 per cent manganese. Walter Ramberg, USA

662. **G. J. Heimerl and D. E. Niles, "Column and plate compressive strengths of aircraft structural materials: Extruded 0-1HTA magnesium alloy,"** *Nat. adv. Comm. Aero. Tech. Note No. 1156*, Jan. 1947, pp. 1–22.

Column and plate compressive strengths of extruded 0-1HTA magnesium alloy were determined, both within and beyond the elastic range, from tests of flat-end H-section columns and from local instability tests of H, Z, and channel-section columns, as part of a program for establishing the column and plate compressive strengths of a number of the alloys available for use in aircraft structures. Tests on aluminum alloys were previously reported.

The results of the present series of tests are presented in the form of curves and charts. Different shapes of cross section, lengths of columns, and dimensions of webs and flanges were tested. Reduced moduli of elasticity, τE_c and ηE_c, are employed in studying the buckling strength of columns and the local instability of plates, respectively. These reduced moduli are defined as the values which must be substituted for the compressive modulus of elasticity E_c in elastic theories for buckling stress, to obtain the experimentally observed values of the buckling stress. Variations of τ and η with stress are reported for each shape of cross section. Curves are also given showing the relation between the actual critical stress and the theoretical critical stress for elastic buckling. Frank Baron, USA

663. **T. D. Northwood, "Sonic determination of the elastic properties of ice,"** *Canad. J. Res. Sec. A*, Mar. 1947, vol. 25, pp. 88–95.

The velocities of three different types of elastic waves were measured in order to determine Young's modulus and Poisson's ratio

for isotropic ice. The longitudinal velocity was obtained by measuring the resonant frequency of the longitudinal vibrations of a bar of the ice. This was done at several temperatures between 0 C and −30 C, and a slight temperature effect was noted. The extensional velocity was determined by measuring the time for a "pulse" to travel through a block of the ice. The Rayleigh wave velocity was determined by measuring the time for a pulse to travel along one surface of a block of ice.

The value obtained for Young's modulus was 9.8×10^{10} dynes per sq cm and for Poisson's ratio was 0.33. These results are compared with previously published values of Young's modulus, the smallest of which was 9.4×10^{10} and the largest of which was 11.2×10^{10} dynes per sq cm. Evan A. Davis, USA

664. L. B. Fonda, "High-temperature disk-forging developments for aircraft gas turbines," *Trans. Amer. Soc. mech. Engrs.*, Jan. 1948, vol. 70, pp. 1–12.

This investigation was started as a result of disk failures in service. Its purpose was to correlate the physical properties, the method of manufacture, and the accuracy of inspection methods with the bursting speed of I-40 turbine disk forgings. Bursting tests were made on 179 turbine-wheel blanks and 6 bucketed turbine wheels at room temperature.

Center ductility was found to be the most important consideration, and proper control of grain flow to be the best method of obtaining it. Unsound forgings, the source of previous difficulties, can be prevented by using extreme care to produce sound ingots and billets. An inspection method, a combination of zyglo and supersonic testing, was developed for detecting any harmful defects, and the inspection results were correlated with bursting tests. M. J. Manjoine, USA

Mechanics of Forming and Cutting Processes

665. Z. M. Rogowsky, with discussion by F. R. Eirich, "Mechanical principles of the screw extrusion machine," *Proc. Instn. mech. Engrs.*, 1947, vol. 156, no. 1, pp. 56–65.

In a theoretical discussion, relationships for the pressure developed, rate of discharge, and drive power required for screw extrusion machines are developed from a dimensional analysis of the mechanics of the process. The analysis is conducted for the extruded material acting in one limit as a Newtonian liquid, and in the other as a Bingham body. Deductions which may lead to improvement of such machines are made, and in particular a criterion for rating screw extrusion machines is suggested.

An interesting discussion of the paper (by Eirich) analyzes the construction of a screw machine for applying high pressures and rates of shear to Newtonian liquids. George Gerard, USA

666. G. Espey and G. Sachs, "Experimentation on tube drawing with a moving mandrel," *J. appl. Mech.*, June 1947, vol. 14, pp. 81–87.

Previous analyses of drawing thin-walled tubes with a moving mandrel are extended and simplified to allow a comparison between experimentation and theory. The drawing stress is derived for two cases: (1) where the flow stress is considered as constant, and (2) where the flow stress varies with progressing reduction.

Several series of tests are reported with variations in: (1) the angle of tapered dies, (2) the mandrel diameter, (3) the metal. The materials used were commercially hard-drawn, and commercially annealed, cartridge brass 0.563 in. OD, 0.032 in. thick, and commercially annealed carbon steel 0.566 in. OD, 0.048 in. thick. The drawing speed was approximately 1 in. per min. Curves are given showing drawn stress as a function of reduction in area, for half die angles of 3, 5, 7, and $17\frac{1}{2}$ deg. The analysis was not carried out for the annealed brass, since the large amount of strain hardening made it difficult to evaluate the flow stress.

The authors conclude that tube drawing with a moving mandrel can be analyzed in much the same manner as other forming and drawing processes. The friction coefficients derived from the analysis are smaller than those previously reported. This is explained by the attention which was paid to polishing the tools, preparing the metal surfaces, and selecting the lubricants, as a result of previous test experience. C. O. Dohrenwend, USA

667. S. Geleji, "Pressing of L- and U-profiles from sheet" (in English), *Publ. tech. Univ. Budapest (Müegyetemi Közl.)*, 1947, no. 1, pp. 14–24.

In this paper the author develops a theory for the forces required to form sheet metal into bent angles or U-shapes.

After taking exception to the accuracy of Schuler's formula for the required bending force, formulas are presented for the bending moments, portion of thickness subjected to elastic deformation, formed radius, total forming load required, etc. Experimental data are presented in chart form and compared to theoretical curves. E. A. Brittenham, Jr., USA

Soil Mechanics; Seepage

668. Alfons Schroeter, "Earth support according to the lattice-wall principle (Erdstützwerke nach dem Gitterwand-Prinzip)," *Bauplan. u. Bautech.*, Oct. 1947, vol. 1, pp. 121–125.

The author discusses the advantages of retaining walls of the lattice-wall type. Terzaghi and Krey have shown that in a densely compacted cohesive filling with some hydraulic pressure, the earth pressure may reach double or treble the Coulomb value, and this effect is enhanced by impact and by the fact that the moment arm is greater than one third of the height of fill. This fact has been evidenced by numerous failures.

The lattice type of retaining wall consists of a relatively thin vertical wall, with a series of horizontal slabs arranged in several stages attached at one end to the wall and anchored by friction in the filling earth at the other end. This system affords great economy of material as compared to gravity walls and to L-shaped walls, as well as to those with stabilizing projections. The author claims the following advantages: (1) Neutralization of the action of abnormally high earth pressures; (2) centralization of the resultant force, considering impact effects; (3) saving of anchoring rods and of pile rafts; (4) setting up of a constantly active retaining force in the place of the anchoring rod, without prestressing; (5) economy. Ch. Széchy, Hungary

669. H. Straub, "The settling of sandy soils of variable density (Das Setzungsverhalten von Sandböden verschiedener Dichte), *Bauplan. u. Bautech.*, Jan. 1948, vol. 2, pp. 21–24.

In this paper the stiffness number E is employed as an index of the settling of sand under loading not resulting in shear failure. E = (unit stress) / (unit strain) of the laterally confined sand mass, is akin to tangential modulus of elasticity, except that permanent as well as reversible strain is involved, without, however, actual frictional movement. E is a function of: (*a*) the state of densification, (*b*) the pressure conditions, (*c*) the roughness of the grain surfaces.

Relative density is more important than absolute porosity. Densification of rounded grains decreased settlement to about

one seventh of the original value, while densification of sharp-edged grains decreased settlement to one third of the original. Increase in pressure at constant density increases E. Observed decrease in settlement for the range of pressure employed varied from 1:2.5 to 1:5.3. Correct use of E therefore involves knowledge of the existing pressure conditions. The greater the roughness of the sand grains, the greater is E for the same porosity.

Improvement of stiffness and decrease in building settlement is furthered more by densification—vibration, impact, etc.—and by admixture of sharp-edged granular materials than by static preloading. Hans F. Winterkorn, USA

670. L. F. Harza, "The significance of pore pressure in hydraulic structures," *Proc. Amer. Soc. civ. Engrs.*, Dec. 1947, vol. 73, no. 10, pp. 1507–1528.

Hydrostatic uplift force depends on both the horizontal area exposed to pore pressure and on the magnitude of the uplift pressure. The author concludes that 100 per cent of the uplift area is effective, in spite of the usual doctrine that this cannot be true because much of the base area must be in contact with the subsoil to support the load. He claims that even the best concrete has voids which communicate with each other, thus giving space for a buoyancy effect, just as if all particles were surrounded by water. He refers to experiments of Terzaghi and Leliavsky in support of these assumptions.

As to the magnitude of the uplift force, the author states that the uplift pressure is not applied at the entrance face, but along the route of seepage, and owing to friction losses it gradually diminishes with the length of the route covered. Actual values of these losses and the role of the viscosity of the fluid are not discussed. (The reviewer notes that while there may be no direct communicating fissure at the base of a hydraulic structure, the underlying rock or other subsoil always contains internal cracks or voids which afford the necessary continuity.)

The author admits that gravity dams with base width of seven tenths of the height could not stand if full pressure and 100 per cent effective uplift area were assumed, but argues that this does not prove the nonexistence of the area, because it is compensated by the loss in the magnitude of the pressure. Finally, he points out the false concept in the dimensioning of round-head buttress dams, where the pressure will actually not be directed towards the buttresses, but, following the shortest seepage route, will exert a considerable cantilever effect. Ch. Széchy, Hungary

Potential or Laminar Incompressible Flow

(See also Revs. 679, 699)

671. L. N. Sretensky, "On the diffusion of a vortex pair" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci.* (*Izv. Ak. Nauk SSSR Ser. tekh. Nauk*), 1947, no. 3, pp. 271–300.

The problem posed is: Given initially two counterrotating vortexes developed only over two small circular areas of radii R in a fluid otherwise at rest, what is the subsequent two-dimensional motion of the incompressible viscous fluid?

A first approximation to the solution is attempted by assuming that instantaneous streamlines coincide at any moment with those corresponding to some free vortex pair moving in an ideal fluid. As a consequence, bipolar co-ordinates and complex variables become the natural and elegant tools. The equation for energy balance simplifies to a linear parabolic differential equation for the stream function f; its solution is laboriously expanded in a series of Legendre polynomials with coefficients exponentially decreasing with the time t (the convergence is slow for small values of t and spoils some of the results). A second assumption is made in order to determine the speed c with which the streamline pattern is moving with respect to the fluid: namely, the resultant of normal pressures across the axis of symmetry between the two vortexes is assumed to vanish. The use of Navier-Stokes' equations then connects c and f.

The results are interpreted primarily in terms of the lateral displacement of the vortex pair, the dissipated kinetic energy, and the vorticity variation. For instance, for initial vortexes of radii $R = 0.5$ cm, 10 cm apart, in water, it appears that during the first 50 sec practically no displacement occurs, while 99.90 per cent of the kinetic energy is already converted to heat. With $R = 1.5$ cm, 92.17 per cent of the kinetic energy is dissipated during the same time. The paper has no translated summary, but the equations are carefully labeled so that it is comparatively easy to follow the argument. M. V. Morkovin, USA

672. R. M. Morris, "The two-dimensional hydrodynamical theory of moving airfoils—IV," *Proc. roy. Soc. London, Ser. A*, Feb. 25, 1947, vol. 188, pp. 439–463.

This is the fourth of a series of papers devoted to the analysis of potential flow about moving cylinders. The stability of small oscillations of a flat-plate airfoil is investigated, including the effect of the wake. Comparison with an analysis omitting the effect of the wake indicates that the inclusion of the effect of the wake may be roughly approximated, in the simpler analysis, by shifting the center of gravity of the element toward the trailing edge by a distance equal to one eighth of the chord length.

Stephen H. Crandall, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 740, 751)

673. A. Fainzilber, "Generalization of the theory of 'mixing lengths' for flow around curved profiles" (in Russian), *Notes Acad. Sci. USSR* (*Doklady Ak. Nauk SSSR*), Nov. 1, 1947, vol. 58, no. 4, pp. 555–558.

The author applies the Prandtl-von Kármán formulas for shearing stress and mixing length to flows with pressure gradient dp/dx along the curved boundary. As a consequence of the approximate equations in the boundary layer, the Prandtl constant-shear assumption $\tau = \tau_0$ is replaced by an assumption of linear variation with distance normal to the wall. Integration of the resulting equations leads to formulas for the velocity profile and skin friction coefficient. Experiments on airfoils by Fage and Faulkner [*Rep. Memo. aero. Res. Comm. Lond.*, no. 1315, 1931] and by Gruschwitz [*Ingen.-Arch.*, 1931, vol. 2, no. 3] provide a satisfactory check and permit the empirical determination of the usual undetermined constant. M. V. Morkovin, USA

674. V. G. Nevzgliadoff, "Application of the phenomenological turbulence theory to flow in pipes" (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), Nov. 1947, vol. 17, pp. 1359–1370.

The author applies the phenomenological theory of turbulence he developed in an earlier paper [*J. Phys. Acad. Sci. USSR* (*Zh. Fiz. Ak. Nauk SSSR*), 1945, vol. 9, pp. 235–243] to the flow of fluids through circular pipes. Some preliminary results for this problem were also given at the end of the earlier paper. After making certain simplifying assumptions the author solves his equations approximately for the mean velocity profile, for the case of both smooth and rough walls. Comparison with Nikuradze's experimentally determined velocity profiles seems to show

exceptionally good agreement. A discussion is also given of the accuracy of his equations and of his approximate method for solving them.

For a short description of the author's approach see Rev. 326, APPLIED MECHANICS REVIEWS, Feb. 1948.

J. V. Wehausen, USA

675. Werner Pfenninger, "Investigations on reductions of friction on wings, in particular by means of boundary-layer suction," *Nat. adv. Comm. Aero. Tech. Memo. No. 1181*, Aug. 1947, pp. 1–152 (transl. from *Inst. Aerodyn. T. H. Zurich*, 1940, no. 13).

This paper deals with the maintenance of a laminar boundary layer up to the trailing edge of a wing. It discusses the causes of transition from laminar to turbulent boundary layer, and the type of airfoil profile favorable for maintaining laminar flow well toward the rear without boundary-layer control. The history of boundary-layer control by suction is given, and the published tests are summarized. Tests are reported of a flat plate with one suction slot, with three suction slots in series, and of the slot flow. Tests were made at zero angle of attack for a wing of symmetrical section, 3.35 per cent thick with eight consecutive slots, to investigate the retention of a laminar boundary layer for effective Reynolds numbers as high as 4.6 million. Tests were also made with a cambered profile 10.5 per cent thick.

There is no doubt that a laminar boundary layer can be maintained by the use of such slots. It should be noted, however, that construction of an actual wing of this type may present formidable problems. The method depends on the existence of laminar flow over the leading edge of the wing. Such laminar flow might be destroyed on a practical wing under service use.

Conrad A. Lau, USA

676. E. Abody-Anderlik, "Investigation of turbulence in parallel, convergent, and divergent channels" (in English), *Publ. tech. Univ. Budapest* (*Müegyetemi Közl.*), 1947, no. 2, pp. 94–109.

Experiments in a small wind tunnel (described briefly) with a movable wall showed that turbulence induced by grids decays more rapidly in a converging stream and less rapidly in a diverging stream than in a stream of constant cross section. This is approximately in accordance with formulas developed from Taylor's decay function, on the assumption that in a stream of changing cross section the fluctuation velocity parallel to the main flow is proportional to the mean velocity, while the fluctuation velocity perpendicular to the main flow is proportional to the square root of the mean velocity.

Isotropic turbulence measurements were not extensive enough to test Taylor's general correlation theory, and showed no agreement with correlation values calculated by von Kármán's method. The isotropic turbulence scale number $(\lambda/\lambda_o)^2$ increased linearly with x/d (x = distance downstream from grid, d = diameter of grid wire) while λ/λ_o decreased linearly with $1/\log Ud$ (U = mean velocity). Benjamin Miller, USA

677. H. J. Allen and G. E. Nitzberg, "The effect of compressibility on the growth of the laminar boundary layer on low-drag wings and bodies," *Nat. adv. Comm. Aero. Tech. Note No. 1255*, July 1947, pp. 1–20.

Equations for the thickness and the Reynolds number of a laminar boundary layer in a compressible fluid are determined for two-dimensional thin profiles and bodies of revolution. Von Kármán's momentum integral relation is used, assuming a Blasius distribution of the velocity through the boundary layer. In addition, the temperature distribution through the boundary layer is assumed to be that given by Emmons and Brainerd ["Temperature effects in a laminar compressible-fluid boundary layer along a flat plate," *J. appl. Mech.*, 1941, vol. 8, pp. 105–110] and the distribution of the surface temperatures is assumed to be the same as that over a circular cylinder as found by Eckert ["Temperature recording in high-speed gases," *Nat. adv. Comm. Aero. Tech. Memo. No. 983*, 1941].

The boundary-layer Reynolds number, which is used as the measure of the boundary-layer stability, is then computed, using the value of the kinematic viscosity occurring at the surface of the body. The calculated results appear to be in good agreement with experiments. Hideo Yoshihara, USA

678. Robert T. Jones, "Effects of sweepback on boundary layer and separation," *Nat. adv. Comm. Aero. Tech. Note No. 1402*, July 1947, pp. 1–8.

In this paper oblique viscous flow is discussed, following the law of stress adopted in the Navier-Stokes' equations. The laminar boundary layer on an oblique flat plate is interpreted by the Blasius law. The results of wind-tunnel tests on a circular wire at different angles of yaw are given. The stability of laminar flow is discussed with reference to other papers.

The known adverse effects of sweepback on the lift and drag of a wing are partially explained by the analysis in this paper.

P. C. Chu, USA

679. G. A. Gourzhienko, "The turbulent flow in diffusers of small divergence angle," *Nat. adv. Comm. Aero. Tech. Memo. No. 1137*, Oct. 1947, pp. 1–77 (transl. from *Cent. Aero-Hydrodyn. Inst.* (*Tsent. Aero-Gidrodin. Inst.*) *Rep.*, no. 462, 1939, Moscow).

Starting with the hydrodynamic equations of Euler and the assumptions of Reynolds, formulas are derived for computing the velocity and pressure distributions in the turbulent flow along and perpendicular to the axis of a diffuser of small cone angle. A test setup is described by means of which the velocity and pressure distributions were measured in two conical diffusers with angles of 1 and 2 deg. Curves comparing the test with the theoretical values show that the assumption of the radial character of the flow holds true to a satisfactory degree for both diffusers investigated.

A first, very approximate attempt is made at estimating, semiempirically, the deviation of the true motion from the radial pattern assumed. This analysis is also based on the assumption that the motion is radial, but emanating from another fictitious source.

The satisfactory agreement obtained between the theoretical and the test results is doubtless a consequence of the smallness of the divergence angles of the diffusers investigated. With further increase in the divergence the effect of the secondary flows will undoubtedly show up to an increasingly greater extent.

Nicholas Di Pinto, USA

680. Brennecke, "Maintaining laminar flow in the boundary layer using a sweptback wing," *Nat. adv. Comm. Aero. Tech. Memo. No. 1180*, Feb. 1948, pp. 1–7 (transl. from *Dtsch. Luftfahrtforsch. Untersuch. Mitt.*, no. 3151, 1944).

To test a conjecture of Lippisch, namely, that sweepback would delay transition, observations of transition location were carried out on a straight and on a 35-deg sweptback wing. The wings had equal chords and identical profiles (NACA 0012-64 at the root) in planes parallel to the stream. Transition was measured

by a dust-precipitation technique. Tests were run at a Reynolds number of 0.56×10^6, based on mean chord. The results indicate no difference at zero incidence, where transition was surprisingly found at about 90 per cent of the chord. However, a rearward shift of about 30 per cent of the chord (from 40 to 70 per cent) occurred at 3 deg incidence, due to sweepback. Apparently there would also be a large effect if the comparison were made at equal lift coefficients.

Since the wing profiles were not the same in sections normal to the leading edge, and since the Reynolds number was not varied, these tests do not afford a good check on modern theories of the yawed laminar boundary layer. W. R. Sears, USA

Compressible Flow, Gas Dynamics

(See also Revs. 677, 678, 680, 688, 689, 691, 693, 694, 695, 697, 698, 710, 714, 719, 728, 729, 742, 747)

681. **B. M. Kiselev, "Calculation of one-dimensional gas flow"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Jan.-Feb. 1947, vol. 11, pp. 177–192.

The first section of this paper presents a survey of the principles of one-dimensional compressible flow of a perfect gas. The theory of flow with variable cross section, with heat exchange, and with sudden contraction and expansion is developed with reference to static pressure and temperature changes.

These basic principles are applied in the second section to the performance of an ejector. Formulas are developed for the pressure and momentum relationships for compressible flow ejectors. Both subsonic and supersonic flow are considered, omitting, however, any shock phenomena. Newman A. Hall, USA

682. **Arthur Kantrowitz, "The formation and stability of normal shock waves in channel flows,"** *Nat. adv. Comm. Aero. Tech. Note No. 1225*, Mar. 1947, pp. 1–51.

Experiments show that shock-free compression channel flows through the speed of sound (converging diverging channels) are unstable, and an analytical treatment of this question, under the assumptions of one-dimensional flow, is presented in the paper.

In first approximation, it is shown that perturbations having the shape of pulses are "trapped" near the throat when moving upstream, i.e., they approach a stationary state corresponding to an asymptotic position. A more accurate treatment shows that expansion pulses, once trapped, die out, while compression pulses grow under the same assumptions. In general, although there is no definite limit between the first phase and the second, it is legitimate to analyze the period of trapping independently of the second phase (growing or dying).

More generally, if a shock wave exists in the channel, it is shown to be unstable in the converging (supersonic) portion of the channel, while it is stable under pulses of given maximum strength in the diverging portion. In the case of pulses stronger than the maximum, the shock wave crosses the throat and moves upstream, which leads the author to suggest long throats as a stabilizing means. Leon Beskin, USA

683. **F. Frankl, "On the problems of Chaplygin for mixed sub and supersonic flows,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1155*, June 1947, pp. 1–32 (transl. from *Bull. Acad. Sci. USSR*, 1945, p. 121).

This paper is concerned with determining the types of boundary problem to which the hodograph equations of two-dimensional potential motion reduce when the flow is of mixed subsonic and supersonic type. Two special problems are considered: (1) The flow of a jet out of a vessel with plane side walls inclined at an angle to each other, and (2) the flow about a wedge when there is a local zone of subsonic velocities ahead of the wedge.

The paper will be of interest primarily to mathematicians since it deals mainly with existence and uniqueness theorems and since no specific techniques are given for solving the practical problems discussed. However, a general method of approach is suggested for handling the two problems mentioned.

Ascher H. Shapiro, USA

684. **Adolf Busemann, "Infinitesimal conical supersonic flow,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1100*, Mar. 1947, pp. 1–16 (transl. from *Dtsch. Akad. Luftfahrtforsch. Schr.*, 1942–1943).

The author defines a conical-flow field as a supersonic flow for which the isobaric surfaces are conical. In this case, the velocity and pressure depend only on the quantities $\xi = x/z$ and $\eta = y/z$, x, y, z being measured relative to the vertex P of the flow, with z in the direction of uniform flow. In terms of ξ and η, the linearized equation $\phi_{xx} + \phi_{yy} - (M^2 - 1)\phi_{zz} = 0$, for the perturbation velocity potential of the conical flow with Mach number M, is of elliptic type within the circle $\xi^2 + \eta^2 = A^2$ (where $A^{-2} = M^2 - 1$) which corresponds to the Mach cone with vertex at P. It is of hyperbolic type outside the circle; the characteristics at points outside are straight lines tangent to this circle.

For points within the Mach cone, the solution of the equation can be expressed in terms of analytic functions of a complex variable by making the transformation $2A/\zeta = \epsilon + 1/\epsilon$, where $\zeta = \xi + i\eta$. Thus the component w of the perturbation velocity parallel to the uniform flow is the real part of an analytic function $w + is = A f(\epsilon)$. The components u, v perpendicular to the uniform flow are expressible in the form $u + iv = -\frac{1}{2} \int \{\epsilon df + df/\epsilon\}$ the function $f(\epsilon)$ being determined by the boundary conditions.

The application of the method to the determination of the conical flow past a circular cone, the tip of a rectangular plate, and a lifting triangle are discussed briefly. Attention is called to the possibility of superposing two or more conical flows with different vertexes, for the purpose of treating more general flow problems such as the flow behind a rectangular plate of finite thickness.

Robert C. F. Bartels, USA

685. **S. V. Falkovich, "A class of Laval nozzles"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Mar.-Apr. 1947, vol. 11, pp. 223–230.

The author develops a linearized solution of the compressible flow equations which are valid in the neighborhood of the sonic singularity. Within the limits of the linearizing approximation a parallel subsonic flow is brought to sonic velocity, establishing a type of convergent divergent nozzle. An initial change of variable $ds = \sqrt{1 - M^2}\, dW/W$ (where W is the radius vector in the hodograph plane) is made, followed by a transformation using bipolar co-ordinates. The resulting linearized equation is solved in terms of hypergeometric functions. Newman A. Hall, USA

686. **Norman F. Smith, "Numerical evaluation of mass-flow coefficient and associated parameters from wake-survey equations,"** *Nat. adv. Comm. Aero. Tech. Note No. 1381*, Aug. 1947, pp. 1–37.

The author continues a series of NACA reports dealing with the determination of the characteristics of subsonic internal flow systems from wake surveys. He specializes on the mass-flow coefficient. Charts showing this coefficient as a function of the static pressure coefficient and the total-pressure-loss coefficient

(both determined from test data) are given in proper scale for isoenergic flow of various Mach numbers. A chart for a correction factor to be used in case mechanical or thermal energy has been added to the flow and another one for converting mass-flow coefficient to inlet velocity ratio are included.

For details of the computing procedure, reference is made to a previous paper by Baals and Mourhess [*Nat. adv. Comm. Aero. adv. conf. Rep. No. L5H27*, 1945]. Wilhelm Spannhake, USA

687. Carl Kaplan, "Effect of compressibility at high subsonic velocities on the moment acting on an elliptic cylinder," *Nat. adv. Comm. Aero. Tech. Note No. 1218*, Mar. 1947, pp. 1–46.

The flow of a compressible fluid about an elliptic cylinder for vanishing thickness can be obtained by the Prandtl-Glauert approximation. To improve this approximation, the stream function may be expressed in the form $\psi = -UY + \psi_1(X, Y) + \psi_2(X, Y) + \psi_3(X, Y) + \ldots$ (U being the undisturbed velocity, X, Y co-ordinates of the flow plane) in which each stream function is considered small compared to the preceding one. By insertion of this expression into the equation of motion

$$\frac{\partial}{\partial X}\left(\frac{\rho_1}{\rho}\frac{\partial \psi}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\frac{\rho_1}{\rho}\frac{\partial \psi}{\partial Y}\right) = 0$$

(ρ_1 being the undisturbed density) equations for the successive stream functions are obtained in terms of the preceding stream functions.

By this method (which leads to some three pages of equations for the second approximation) expressions for the lift and moment are obtained. The resulting equations are numerically evaluated and expressed in table and graphical form for 4 different ellipse thicknesses and 14 different subsonic Mach numbers. In addition, the movement of the center of pressure caused by compressibility effects is computed. Howard W. Emmons, USA

Aerodynamics of Flight; Wind Resistance

(See also Revs. 672, 675, 677, 678, 680, 718, 720, 722, 727, 734, 742, 743)

688. John C. Evvard, "Distribution of wave drag and lift in the vicinity of wing tips at supersonic speeds," *Nat. adv. Comm. Aero. Tech. Note No. 1382*, July 1947, pp. 1–33.

In this paper the method of point-source distribution for solving the boundary problem of flows past wing surfaces is extended to cases where the foremost Mach wave lies ahead of the wing boundary. A point-source distribution arranged merely on the wing surfaces gives a solution for flat wings at an angle of attack only in case the leading edge lies ahead of the Mach wave. Generally, without regard to the situation of the foremost Mach wave, this method is valid only for symmetric (and very slender) bodies, at zero angle of attack. In case the leading edge is behind the Mach wave, point sources distributed only over the top and bottom surfaces would, for an angle of attack, produce a physically impossible pressure discontinuity in the flow between the airfoil leading edge and the Mach wave, due to the asymmetry of the solutions for the upper and lower half space.

The author removes this difficulty by arranging external point sources just in the region between the leading edge and the Mach wave. He determines their strength so that, in combination with the point-source distribution on the wing surfaces, they do not produce any discontinuity of the velocity potential and, consequently, of the pressure in the flow between the Mach wave and leading edge. Furthermore, he finds that in special cases it is not necessary to calculate the strength of the external point sources explicitly. He shows that by using an oblique co-ordinate system, the axes of which lie parallel to the Mach waves, the contribution of the external point sources to the potential on the wing surface may be replaced by an equivalent integration over a portion of the wing surface.

Several examples for applying the method are given. The possibility of superimposing several solutions is discussed. Wilhelm Spannhake, USA

689. J. C. Evvard and L. R. Turner, "Theoretical lift distribution and up-wash velocities for thin wings at supersonic speeds," *Nat. adv. Comm. Aero. Tech. Note No. 1484*, Nov. 1947, pp. 1–49.

An integral equation defining the slope of the streamlines in the disturbed supersonic flow between the wing boundary and the foremost Mach wave is solved to determine either the surface pressures on the wing or the up-wash velocity near the wing tips.

The explicit solution is of course complicated for all but the simplest boundary conditions, because of the singularities on the characteristic surface or Mach cone. However, this is precisely the difficulty that had already been removed for the general case by Hadamard when he introduced the concept of "the finite part" of the integral. Consequently the numerical solution developed by the authors is analogous to Hadamard's method [see Webster, "Partial differential equations of mathematical physics," pp. 277–280], but is not developed as far as the more powerful general method originally given by Hadamard. E. V. Laitone, USA

690. L. Sternfield, "Effect of product of inertia on lateral stability," *Nat. adv. Comm. Aero. Tech. Note No. 1193*, Mar. 1947, pp. 1–12.

The author points out the importance of the product of inertia terms in the lateral equations of motion for airplanes having relatively high density factors (ratio of mass of the airplane to mass of a characteristic volume of air) such as are encountered on modern high-speed high-altitude airplanes. The product of inertia terms have usually been neglected in past analyses, because earlier studies on older airplanes had shown their effect to be small.

Analyses are given in this paper for an experimental fighter airplane, a hypothetical supersonic fighter, and a free flight wind-tunnel model. In all cases inclusion of the product of inertia terms produced a stabilizing shift of the oscillatory stability boundaries. Some free flight test results are cited to verify the conclusions of the analysis. A. H. Flax, USA

691. W. Jacobs, "Pressure-distribution measurements on unyawed sweptback wings," *Nat. adv. Comm. Aero. Tech. Memo. No. 1164*, July 1947, pp. 1–30 (transl. from *Dtsch. Luftfahrtforsch. Untersuch. Mitt.*, no. 2052, 1943).

The report presents comprehensive pressure-distribution measurements on four sweptback wings with angles of sweepback of 0, 15, 30, and 45 deg respectively. All the wings had constant chord and aspect ratio. The measurements were taken in a wind tunnel at an air speed of 90 mph and at the Reynolds number 4.2×10^6. The pressure measurements were supplemented by force measurements.

The integrated data are presented graphically to show the spanwise distribution of section lift coefficient, the relation between wing lift coefficient and angle of attack, and the relation between the spanwise location of the center of lift and the angle of sweepback. It is concluded from these results that the center of the lift distribution on each semiwing section tends to move toward the wing tip with an increase in the angle of sweepback. The experimental results are compared with theoretical calculations based on the methods of J. Weissinger [*Nat. adv. Comm.*

Aero. Tech. Memo. No. 1120, Mar. 1947] and H. Multhopp ["The application of wing theory to the mechanics of flight in unsymmetrical flows," *Lilienthal Ges. Luftfahrtforsch. Ber.*, no. 52, 1939]. This comparison tends to indicate that Weissinger's method is superior to Multhopp's. Robert C. F. Bartels, USA

692. **M. D. White, H. Lomax, and H. L. Turner, "Sideslip angles and vertical tail loads in rolling pull-out maneuvers,"** *Nat. adv. Comm. Aero. Tech. Note No. 1122*, Apr. 1947, pp. 1–35.

The equations of lateral stability are solved for the sideslip angle of an airplane executing a rolling pull-out maneuver. The results are given in the form of design charts for typical ranges of the lateral stability derivatives. In addition, simple approximations are given, which will generally be adequate for preliminary design. Provision for nonlinear derivatives is made.

The results are applied to the calculation of vertical tail loads, neglecting sidewash, etc., and it is stated that such loads will generally exceed those based on other criteria. Flight test measurements verify all results for typical configurations.

John W. Miles, USA

693. **Harvey H. Brown and Lawrence A. Clousing, "Wing pressure-distribution measurements up to 0.866 Mach number in flight on a jet-propelled airplane,"** *Nat. adv. Comm. Aero. Tech. Note No. 1181*, Mar. 1947, pp. 1–58.

Chordwise pressure distributions were determined at four spanwise positions on one wing of a jet-propelled airplane. Data were taken during accelerated and unaccelerated flight at several Mach numbers. The flight results are shown to be in good general agreement with related wind-tunnel data, although discrepancies between the model and full scale production airfoil contour produced measurable differences between the flight and tunnel distributions. Edward N. Bowen, USA

694. **A. Kahane and Lester Lees, "The flow at the rear of a two-dimensional supersonic airfoil,"** *J. aero. Sci.*, Mar. 1948, vol. 15, pp. 167–170.

The angle of flow from the trailing edge of a two-dimensional airfoil moving at supersonic speeds is calculated for a nonisentropic flow in the absence of shock-wave detachment. It is shown that for a flat-plate airfoil a down-wash occurs at Mach numbers less than 1.28, while up-wash occurs at higher Mach numbers. This result is at variance with a previous treatment [M. J. Lighthill, *Rep. Memo. aero. Res. Comm. Lond.*, no. 1930, 1944], which appears to be in error. In any event the deviation of the angle is small, being always less than 0.06 deg for an angle of attack of 5.7 deg. For airfoils of finite thickness the flow angle in the wake is increased. A formula for the trailing-edge flow angle for an airfoil of finite thickness is included.

The mechanism by which the flow returns to the free-stream direction is also discussed. It is indicated that the interaction of the expansion and compression waves which emanate from each surface of the airfoil is responsible for the ultimate return of the flow to that of the free stream. H. Julian Allen, USA

695. **Herbert S. Ribner, "The stability derivatives of low-aspect-ratio triangular wings at subsonic and supersonic speeds,"** *Nat. adv. Comm. Aero. Tech. Note No. 1423*, Sept. 1947, pp. 1–34.

The stability derivatives of low-aspect-ratio triangular wings are determined on the assumption that the velocity potentials in planes at right angles to the long axis of the airfoils can be treated as two-dimensional potentials. In all the types of motion considered, the primary problem is then to determine the two-dimensional surface potential for the flow about a straight line with specific boundary conditions.

The pressure differences between the upper and lower surfaces due to downward acceleration, pitching, rolling, sideslipping, and yawing are obtained from the approximate form of Bernoulli's law assuming small disturbances, for wings with and without dihedral. The results apply to an isolated triangular wing in the limiting case of aspect ratio approaching zero, with an aspect ratio of 0.5 estimated as the upper limit of utility. With the exception of the transonic range, the stability derivatives are expected to apply at both subsonic and supersonic speeds up to a limiting speed at which the triangle is no longer narrow in comparison with the Mach cone from its vortex.

W. F. Milliken, Jr., USA

696. **Paul E. Purser and Charles B. Cook, "Collection and analysis of hinge-moment data on control-surface tabs,"** *Nat. adv. Comm. Aero. Tech. Note No. 1113*, Apr. 1947, pp. 1–31.

Tabs are used on most aircraft to provide the final adjustment of control-surface forces, and the hinge moment of a tab about its own hinge axis is an important design factor. This report lists 27 references and discusses their coverage of data for tab design. The information is plotted, and is extended by unpublished results of tests by the NACA. A formula for aspect-ratio correction developed from lifting line theory is given.

The greatest lack of information at present is at high Mach numbers; some data taken at high speeds are available and are presented, but they are so limited as to be practically useless for design purposes. W. C. Johnson, Jr., USA

697. **Doris Cohen, "The theoretical lift of flat sweptback wings at supersonic speeds,"** *Nat. adv. Comm. Aero. Tech. Note No. 1555*, Mar. 1948, pp. 1–77.

The lift of thin sweptback wings in a supersonic flow is computed by the author, using the method of superposition of linearized conical flows. The cases which are treated are as follows:

1 The leading and trailing edges both lie ahead of their respective Mach lines.

2 The leading edge, but not the trailing edge, is swept behind the Mach lines.

3 The leading and trailing edges are both swept behind their respective Mach lines. (The Mach line from the trailing edge does not intersect the leading edge.)

In making the tip corrections a new approach is made, consisting essentially in superimposing an infinite number of overlapping conical flows. This method is also used to fulfill approximately the Kutta condition in the third case.

Hideo Yoshihara, USA

698. **Kenneth Margolis, "Effect of chordwise location of maximum thickness on the supersonic wave drag of sweptback wings,"** *Nat. adv. Comm. Aero. Tech. Note No. 1543*, Mar. 1948, pp. 1–30.

The linearized supersonic wing theory is used to compute the zero-lift wave drag of a family of sweptback wings having no taper. The planform considered has a 60-deg sweep angle and an aspect ratio of two. The airfoil section considered is a double-wedge section with the maximum thickness line parallel to the leading and trailing edges.

It is shown that the minimum wave drag of this family of wings is produced if the maximum thickness line is at the mid-chord position. This result is found both for supersonic and for subsonic leading edges. H. J. Stewart, USA

699. Arthur L. Jones and Loma Sluder, "An application of Falkner's surface-loading method to predictions of hinge-moment parameters for sweptback wings," *Nat. adv. Comm. Aero. Tech. Note No. 1506*, Feb. 1948, pp. 1–17.

The application of Falkner's method to straight and sweptback wings of various aspect ratio is considered, particular attention being given to the prediction of the hinge-moment parameters. The method consists of simulating the finite wing by a distribution of vortex loops, the intensity of these loops being determined in effect by a simultaneous solution for the down-wash at an equal number of control points. The boundary conditions are satisfied only at the control points. The method is thus in essence approximate, the degree of approximation depending on the energy of the computer.

As has previously been shown, over-all results such as $dC_L/d\alpha$ can be obtained with good accuracy. The present report shows, however, that the loadings obtained by this method are not sufficiently accurate to obtain hinge-moment coefficients. The authors conclude that the discrepancies are primarily due to the viscosity effect neglected in the theory. C. B. Smith, USA

700. A. Gessow and G. C. Myers, Jr., "Flight tests of a helicopter in autorotation, including a comparison with theory," *Nat. adv. Comm. Aero. Tech. Note No. 1267*, Apr. 1947, pp. 1–23.

Flight test data of a Sikorsky HNS-1 (YR-4B) helicopter in autorotation are presented. Rotor drag-lift ratios and related parameters were derived from autorotation flight data, and performance at standard sea-level conditions was calculated. Good agreement between theoretical and experimental autorotation performance was obtained when the theoretical calculations were based on a profile-drag polar corresponding to a "rough" airfoil section. The authors claim that significant improvement in gliding performance appears possible with improved blade contour and surface condition. S. W. Yuan, USA

701. Alfred Gessow, "Effect of rotor-blade twist and planform taper on helicopter hovering performance," *Nat. adv. Comm. Aero. Tech. Note No. 1542*, Feb. 1948, pp. 1–26.

A theoretical calculation of the hovering performance of helicopter rotors having various combinations of twist and planform taper is made with the aid of strip-analysis procedure similar to that used in propeller analysis. The comparison is based on rotors of equal solidity, the solidities of the tapered blades being computed by means of an equivalent chord.

An increase in thrust of about 5 per cent over rectangular blades is indicated if a combination of -8 to -12 deg linear twist and a taper ratio of root chord to tip chord of 3 are used. An additional 2 per cent gain in thrust is shown for a nonlinear optimum combination of twist and taper. The use of moderately lower solidity and partial instead of full taper does not significantly affect these results. S. W. Yuan, USA

702. H. O. Ankenbruck and M. O. McKinney, Jr., "Generalized performance comparison of large conventional, tail-boom, and tailless airplanes," *Nat. adv. Comm. Aero. Tech. Note No. 1477*, Oct. 1947, pp. 1–84.

Performance of each of these three airplane types is analyzed for a total power of 21,000 and 42,000 bhp, aspect ratio 10, and load factor 4, assuming the same directional stability in each case and maximum lift coefficients of 2.4, 2.4, 2.0, respectively. The computation of the performance characteristics (top speed, ground run, rate of climb, ceiling, and range) is along conventional lines. The main difficulty here is to make a close estimate of the ratio of structural weight to gross weight, which is of paramount importance for the range. Special attention is therefore given to possible deviations of this ratio from its estimated value.

Results are first presented in the form of composite selection charts giving curves of constant speed, range, and take-off distance, in a plot of power loading versus wing loading. While definite superiority of the tailless and slight superiority of the tail-boom airplane are apparent in these plots, a better comparison can be obtained from charts presented which give one of the quantities speed, rate of climb, take-off distance, ceiling versus number of passengers (or bomb load) at *given range* and *given landing speed*. These charts permit the following conclusions: *Tailless airplanes* have better performance characteristics when designed as bombers or *long-range* transports. As *short-range* transports with high wing loadings, conventional airplanes perform better. Tail-boom airplanes cannot compete in any of the types of missions considered. G. Kuerti, USA

703. Wilbur L. Mayo, "Solutions for hydrodynamic impact force and response of a two-mass system with an application to an elastic airframe," *Nat. adv. Comm. Aero. Tech. Note No. 1398*, Aug. 1947, pp. 1–42.

A theoretical method is presented whereby the influence of the elastic behavior of the airframe structure on the hull of a seaplane may be determined for the condition of landing impact. The dynamic bending of wings during impact is considered by reducing the fundamental mode to an equivalent two-mass system.

Time histories of the hydrodynamic force and structural response are given for wide ranges of mass distribution and ratio of natural period to the period of the impact. By use of nondimensional coefficients the results are made applicable to various combinations of velocity, weight, dead-rise angle, and fluid density. The shape of the force-time history may be considerably changed through consideration of dynamic elasticity of the airframe structure, and the maximum hydrodynamic force might either be reduced or increased. Ernest G. Stout, USA

704. Garry C. Myers, Jr., "Flight measurements of helicopter blade motion with a comparison between theoretical and experimental results," *Nat. adv. Comm. Aero. Tech. Note No. 1266*, Apr. 1947, pp. 1–35.

Rotor-blade motion of a conventional single-rotor helicopter was recorded photographically during flight. The measurements include flapping motion, in-plane motion, and blade twisting and bending, at tip-speed ratios ranging from 0.12 to 0.25. Satisfactory agreement was found between the test results and corresponding theoretical values.

The measured data, given in the form of Fourier series coefficients, may serve as a guide in the estimation of control displacement for trim and static-stability determination, and in the design of the rotor hub. S. W. Yuan, USA

705. Herbert W. Talkin, "Charts showing relations among primary aerodynamic variables for helicopter performance estimation," *Nat. adv. Comm. Aero. Tech. Note No. 1192*, Feb. 1947, pp. 1–51.

This paper presents charts, based upon conventional helicopter theory, showing the relation between design variables and aerodynamic performance of helicopter rotors, as an aid to estimating the performance of this type of aircraft. Performance conditions considered include hovering, level flight, climb, and ceiling. The theory and methods used in the preparation of the charts are given in an appendix. John E. Goldberg, USA

706. **W. Lewis, D. B. Kline, and C. P. Steinmetz, "A further investigation of the meteorological conditions conducive to aircraft icing,"** *Nat. adv. Comm. Aero. Tech. Note No. 1424*, Oct. 1947, pp. 1–26.

This may be regarded as a supplement to a previous report of William Lewis [*Nat. adv. Comm. Aero. Tech. Note No. 1393*, Aug. 1947]. The data from the 1946–1947 observations on liquid water content, temperature, and mean effective drop diameter are shown to be consistent with values previously proposed for maximum icing conditions.

Data on drop-size distribution as obtained by the rotating-cylinder method are also consistent with measurements previously made. However, they were found to be inconsistent with data on drop-size distribution derived from the ratio of the maximum diameter to the mean effective diameter, when the maximum diameter was calculated from the area of impingement on a stationary cylinder. One possible explanation lies in the effect of the acceleration in the flow of air around the fuselage in locally modifying the water content, drop-size distribution, and velocity, at the points where the rotating cylinders are exposed. The relation between temperature and maximum liquid water content in layer clouds is also discussed.

L. J. Tison, Belgium

707. **F. S. Malvestuto, Jr., and L. J. Gale, "Formulas for additional-mass corrections to the moments of inertia of airplanes,"** *Nat. adv. Comm. Aero. Tech. Note No. 1187*, Feb. 1947, pp. 1–28.

In stability investigations of airplanes or dynamically scaled models it is necessary to know the true moments of inertia of the model or plane. To obtain these correctly, the experimental moments of inertia, determined by the pendulum method, must be corrected for the effect of the surrounding air.

Formulas are presented for the evaluation of the additional-mass and moment-of-inertia corrections. A correlation of the values determined by this method with experimental data from tests on 40 models indicates that these formulas are satisfactory.

W. Olszak, Poland

708. **Powell M. Lovell, Jr., "The effect of wing-bending deflection on the rolling moment due to sideslip,"** *Nat. adv. Comm. Aero. Tech. Note No. 1541*, Feb. 1948, pp. 1–18.

A method is presented for calculating the effect of wing flexibility on the rolling moment due to sideslip for wings of various aspect ratios and taper ratios, when different shapes of the bending-deflection curve are assumed. The shape of the deflection curve is shown to be unimportant, the main factor being the amount of wing-tip deflection.

The effect is found to be appreciable on typical airplanes. For example, the rate of change of rolling moment coefficient with sideslip angle of a typical hypothetical bomber is increased by one third for level flight, and is almost doubled at the limit load factor. A method is also given by which the effective dihedral may be modified at the design stage to take the wing bending into account. The effect is important principally in the design of large low-load-factor airplanes. E. Arthur Bonney, USA

709. **Benjamin Milwitzky, "A generalized theoretical and experimental investigation of the motions and hydrodynamic loads experienced by V-bottom seaplanes during step-landing impacts,"** *Nat. adv. Comm. Aero. Tech. Note No. 1516*, Feb. 1948, pp. 1–60.

This gives a theoretical method of analysis of the motions and hydrodynamic impact loads experienced by V-bottom seaplanes during step landings. It is shown that the motion and time characteristics of a landing impact may be represented by dimensionless variables designated as load-factor coefficient, draft coefficient, time coefficient, and vertical-velocity ratio. These variables are governed by the magnitude of the approach parameter, which depends only on the trim and flight-path angle at the instant of initial contact.

The results of the study are presented in the form of dimensionless curves which may be used to predict the behavior of V-bottom seaplanes at all instants during an impact as well as at the particular instants of maximum acceleration, maximum draft, and rebound. The theoretical results are extensively correlated with experimental data. Ernest G. Stout, USA

710. **Charles W. Frick, Jr., "Application of the linearized theory of supersonic flow to the estimation of control-surface characteristics,"** *Nat. adv. Comm. Aero. Tech. Note No. 1554*, Mar. 1948, pp. 1–56.

The characteristics of aircraft control surfaces in supersonic flight cannot be determined completely without rather extensive tests, but theoretical investigations have great qualitative value. In this report the linearized equation for the velocity potential is used to calculate the characteristics of control surfaces with hinge lines and trailing edges either inside or outside the Mach cone. The total lift corresponding to the deflection of such a surface is taken as the sum of a basic lift and an induced lift, the latter being the correction for the finite extent of the wing. Pressure distributions are obtained from known solutions of the linearized equation.

The method is applied to several specific control surfaces (elevators and ailerons), for which curves are presented showing the spanwise distribution of pressure coefficient, lift coefficient, and hinge-moment coefficient, as well as the chordwise distribution of pressure coefficient. Integrated values of the coefficients are also tabulated. C. W. Smith, USA

711. **Melvin Sadoff and Lawrence A. Clousing, "Measurements of the pressure distribution on the horizontal tail surface of a typical propeller-driven pursuit airplane in flight—III. Tail loads in abrupt pull-up push-down maneuvers,"** *Nat. adv. Comm. Aero. Tech. Note No. 1539*, Feb. 1948, pp. 1–61.

Measurements of the pressure distribution on the horizontal tail of a pursuit airplane in an abrupt pull-up push-down maneuver were made. These experimental results are shown to be in good agreement with theoretical results given. Particularly good agreement is obtained if the observed elevator deflections are used in the theoretical analysis.

Comparisons are carried out for the tail load distribution, tail root bending moments, airplane lift coefficient, and the over-all acceleration factor. The theory used is the conventional, linearized, small disturbance maneuverability theory.

H. J. Stewart, USA

712. **J. P. Campbell and C. L. Seacord, Jr., "The effect of mass distribution on the lateral stability and control characteristics of an airplane as determined by tests of a model in the free-flight tunnel,"** *Nat. adv. Comm. Aero. Rep. No. 769*, 1943 (publ. in 1947), pp. 1–13.

This paper deals with an experimental study of the effects of increased lateral and rolling moments of inertia on the lateral stability of aircraft. The tests were made on models of airplanes in the NACA free-flight wind tunnel. Moments of inertia were varied from the normal value for dynamically similar models, to

five times this value. The increased moments of inertia were found to cause reductions in oscillatory lateral stability; this effect was most serious for high values of wing dihedral, but was less important for small or negative dihedral. Test results agreed in general with theory.

An attempt was made to classify types of instability as dangerous or not, depending upon whether it was possible for a pilot to fly the model in the tunnel with external controls. In the opinion of the reviewer this was not valid, since the time scale of the model was one third that of full scale, while the time scale of the pilot, considered as a servomechanism, could not be similarly reduced. A. H. Flax, USA

713. B. Maggin and C. V. Bennett, "Flight tests of airplane models with a 42 and a 62-deg sweptback wing in the Langley free-flight tunnel," *Nat. adv. Comm. Aero. Tech. Note Nos. 1287* (for 42 deg) *and 1288* (for 62 deg), May 1947, pp. 1–30 and 1–24.

Low-speed stability and control characteristic tests of airplane models having 42 and 62-deg sweptback wings are reported and analyzed. Force measurements and tuft studies were used to determine the static stability and control and the wing-stall characteristics of the models. Power-off flight tests were conducted to study the longitudinal and lateral dynamic stability characteristics.

For both angles of sweepback an interesting longitudinal instability at moderate lift coefficients was disclosed which had not been experienced previously in free-flight tunnel investigations. Stability boundaries for lateral oscillations were calculated for comparison with the free-flight tests; the disagreement found for 62 deg sweepback indicated a need for more experimental data on the rotary stability derivatives of highly swept wings.

Arthur L. Jones, USA

714. Chieh-Chien Chang, "A simplified method of obtaining drag of a high-speed body from wake surveys," *J. aero. Sci.*, Feb. 1948, vol. 15, pp. 123–127.

This presents the derivation of a simplified method of obtaining drag of a high-speed body from wake surveys, and illustrates its application to the analysis of flight test data.

The integrand of Jones' drag integral equation is separated into the product of two functions, each of which can be conveniently plotted, making the Pitot-traverse method practical and easy to apply in determining drag at subsonic speed. These two functions are plotted and an application of the method to analysis of flight test data is shown. Application to supersonic flow is also discussed and limitations and suggestions are indicated.

The paper concludes with a discussion of the limitations and justifications of four assumptions made in the analysis: (1) Stagnation temperature is constant along a streamline; (2) air behaves as a perfect gas; (3) velocity fluctuations in the wake are small compared to local mean velocity as calculated from the measured time mean pressure; (4) the flow in the wake is isentropic along the streamline downstream of the measuring station.

H. R. Gillespie, Jr., USA

715. Robert A. Mendelsohn, "A semigraphical method of computing stick forces for spring-tab controls having nonlinear hinge-moment characteristics," *J. aero. Sci.*, Feb. 1948, vol. 15, pp. 107–112.

A proposed semigraphical method for computing stick forces for spring-tab controls, making use of aileron and tab characteristics from wind-tunnel tests, is applied to the case of an assumed airplane with an assumed aileron and tab linkage. A comparison of the results with those obtained by the assumption of linear aileron and tab characteristics indicates that this assumption may be greatly in error at high rates of roll.

Louis Landweber, USA

716. Thomas A. Toll, "Summary of lateral-control research," *Nat. adv. Comm. Aero. Tech. Note No. 1245*, Mar. 1947, pp. 1–193.

In this paper the NACA's work on lateral control is reviewed in the light of modern requirements. Included are discussions of the bases on which control effectiveness are judged, the factors involved in obtaining this effectiveness, and the data available for designing such systems.

This work differs from previous summaries in that proper weight is given to such factors as aerodynamic balance, flexibility of the wing, flutter, and control lag—points which the advent of high speed has made of great importance.

The student will find the report instructive. The practicing engineer will find the compilation of data convenient. All persons concerned with the design and operation of airplanes will find it of interest. C. B. Smith, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 708)

717. Martin Goland and Y. L. Luke, "The flutter of a uniform wing with tip weights," *J. appl. Mech.*, Mar. 1948, vol. 15, pp. 13–20.

The authors develop an exact solution of the differential equations controlling the flutter of a straight beam wing with uniform inertia, stiffness, and aerodynamic characteristics (the latter being based on strip theory), which carries concentrated weights representing a fuselage and wing-tip fuel tanks. A discussion of methods for substantially simplifying the accessory computations is included.

Application of this theory to a fighter-wing representation with two different chordwise tip-weight locations yields flutter speeds which are respectively 0.8 and 6 per cent lower than the results of a conventional two-degree of freedom Rayleigh-type analysis involving the uncoupled modes. The flutter frequencies are 1.4 per cent higher than the Rayleigh-type approximations in both cases. Comparison of the flutter modes obtained in both ways demonstrates the consequences of the absence, in the Rayleigh approximations, of spanwise phase shift, and of readjustment of the amplitude distributions.

It is stated that it is not possible at present to predict the direction of the errors in the conventional analysis, which are apparently sometimes quite appreciable.

J. E. Greidanus, Holland

718. E. Reissner, "Effect of finite span on the air load distributions for oscillating wings—I. Aerodynamic theory of oscillating wings of finite span," *Nat. adv. Comm. Aero. Tech. Note No. 1194*, Mar. 1947, pp. 1–39.

The equations for the distribution of air load on a harmonically oscillating wing of finite span are formulated on the basis of the theory of the lifting surface. This results in a two-dimensional integral equation relating the vertical velocity component at the airfoil to the distribution of vorticity on the airfoil and in the wake. On the assumption that the aspect ratio is not too small, this equation is simplified to a form similar to that obtained in the two-dimensional case, but with an additional term arising

from the effect of the spanwise variation of circulation. This leads to a one-dimensional integral equation for the spanwise distribution of circulation. This equation must be solved for the particular wing deflection functions of interest.

The forces and moments on the wing are evaluated in terms of the circulation distribution function. It is found that the lift and moment functions are similar to those of the usual two-dimensional theory except that the Theodorsen unsteady lift function $C(k)$, where k is the reduced frequency, is modified by the addition of a correction term which depends upon the ratio of the three-dimensional to the two-dimensional circulation function for each section. This ratio is obtained from the solution of the one-dimensional integral equation for circulation function, and depends upon the wing planform, reduced frequency, and wing deflection form. The correction term to be added to Theodorsen's $C(k)$ is found to be the same for lift, moment, and hinge moment for wings, flaps, and tabs.

The author suggests that the approximate theory developed be called the lifting strip theory, to distinguish it from lifting line theory and from lifting surface theory. The principal difference between this theory and lifting line theory is that in this theory the down-wash produced by the trailing vortexes varies along the chord, while it is constant in the lifting line theory. The lifting strip theory does, however, reduce to the lifting line theory in the limiting case of zero frequency. A. H. Flax, USA

719. **I. E. Garrick and S. I. Rubinow, "Flutter and oscillating air-force calculations for an airfoil in a two-dimensional supersonic flow,"** *Nat. adv. Comm. Aero. Tech. Note No. 1158*, Oct. 1946, pp. 1–65.

This paper clearly integrates, organizes, and extends the theoretical results obtained in the investigations of Possio, Borberly, Temple, and Jahn, pertaining to the small oscillations of an airfoil at supersonic speeds. These investigations are based upon the assumption of small perturbations of the nonsteady velocity field relative to the uniform velocity, thereby leading to the linearization of the differential equation for the velocity potential. Accordingly the airfoil is considered to be very thin, oriented at a small angle of attack, and the flow is assumed to be irrotational and nondissipative.

The paper contains useful numerical tables of aerodynamic force coefficients which are developed for Mach numbers greater than unity. Results of a supersonic bending torsion wing-flutter analysis are given. The very interesting case of one degree of freedom torsional supersonic flutter is discussed.

The expression for the forces and moments in the limiting case where the frequency of the airfoil approaches zero are obtained in a useful form. These can be directly applied to the calculation of wing-divergence and control-surface reversal phenomena at supersonic speeds. Paul Lieber, USA

720. **Abbott A. Putnam, "An improved method for calculating the dynamic response of flexible airplanes to gusts,"** *Nat. adv. Comm. Aero. Tech. Note No. 1321*, May 1947, pp. 1–22.

The author replaces the airplane wing-fuselage system by a simple beam and substitutes an empirical forcing function for the action of the gust. By considering only the first mode in wing bending and a constant aerodynamic damping coefficient he reduces the integro-differential equations to two simultaneous linear differential equations with constant coefficients. The solution yields time histories of wing-tip and fuselage accelerations and of the wing-tip deflections.

Although the method represents considerable simplification over that given by Pierce [*Nat. adv. Comm. Aero. Tech. Note No. 1320*, 1947], the care required in determining coefficients and the need for carrying six or eight significant figures make the gain appear slight. Difficulty is also experienced in determining a "static" wing deflection for use in obtaining the percentage of dynamic under or over stress. Philip Donely, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 664, 700, 701, 704, 705)

721. **H. Parkus, "Torsional vibrations of propellers (Drillschwingungen von Luftschraubenblättern),"** *Öst. Ingen.-Arch.*, July 1947, vol. 1, nos. 4 and 5, pp. 296–302.

The usual analysis is extended to include the influence of the centrifugal force as well as the effect of the prevention of warping of the blade cross sections close to the hub. If x denotes the elastic axis of the blade and z the propeller axis, the only centrifugal moment of importance is that due to the component parallel to the yz-plane of the centrifugal force, which is figured as if the mass were concentrated along the x-axis. This is computed in terms of the blade angle and of the angle of twist ϑ, and enters the equation of motion along with the moments of the inertia forces.

The restraint on warping is approximately taken care of by an additional impressed moment, introduced by Kappus, of $-E\frac{\partial}{\partial x}\left(J_w\frac{\partial^2\vartheta}{\partial x^2}\right)$, where $J_w=\int_F \varphi^2 dF$ (F being the cross section and φ the torsion function valid for unrestrained warping). This method was originally conceived for open profile shapes with thin walls, but the author shows that it may also be applied advantageously to propeller cross sections by carrying out the approximate solution in a case for which the correct solution is known.

The correction due to the additional terms is found to be insignificant in the case of rotor blades, while for an average wooden propeller of fundamental torsional (circular) frequency of 450 sec^{-1} the correction is about $+50\,\text{sec}^{-1}$.

An approximate solution which assumes constant blade cross section is tabulated, giving numerical values of the frequency correction in terms of a parameter depending on J_w, J_ϑ and L and the elastic constants, where L is the effective blade length and J_ϑ is the torsional stiffness of the blade. G. Kuerti, USA

722. **Eugene Migotsky, "Full-scale investigation of the blade motion of the PV-2 helicopter rotor,"** *Nat. adv. Comm. Aero. Tech. Note No. 1521*, Mar. 1948, pp. 1–40.

An experimental investigation, effectively at about two thirds of full scale, was made of the PV-2 three-bladed helicopter rotor at the Langley large-scale wind tunnel. The blade motions were recorded with a motion-picture camera rotating with the rotor hub. The tests were conducted under conditions corresponding to trim about a point on the shaft axis nearly four feet below the flapping hinge.

The results, based on measurements at the 0.75 radius station of a blade, are given by charts showing the variation of the harmonic coefficients of the total flapping and of the total feathering angles with the tip-speed ratio μ (varying from about 0.1 to 0.2), mean pitch angle A_0, and rotor-shaft angle α_{ST}. The coefficients are also presented as functions of thrust coefficient C_t and useful drag-lift ratio $(D/L)_u$ (which may replace A_0 and α_{ST} as parameters).

According to the measurements, the flapping motion was very small, the coefficients of the first harmonics being less than

1 deg, and the coefficients of the second harmonics less than 0.25 deg. For the feathering motion the first-harmonic coefficients were larger, and increased in magnitude with both μ and C_T, but the second harmonics were negligible. For a given μ and a given C_T, the flapping and feathering coefficients were nearly constant with $(D/L)_a$.

Comparison is made between these measurements and theoretical calculations by Bailey for an "equivalent" (as defined by Locke) nonfeathering rotor. On the whole, there appeared to be fairly good agreement between theory and experiment. It is pointed out, however, that in regard to the lateral component of equivalent flapping, better agreement can be obtained by assuming an induced down-wash which is not constant but increases linearly from the front to the rear of the disk.

Morris Morduchow, USA

723. W. Byron Brown, "Friction coefficients in a vaneless diffuser," *Nat. adv. Comm. Aero. Tech. Note No. 1311*, May 1947, pp. 1–16.

Experimental investigations were made of the flow in the diffuser surrounding a centrifugal impeller. Friction coefficients for three constant-area vaneless diffusers were determined from static pressure and total pressure surveys taken at several radii, and from the usual over-all measurements of temperature, pressure, and airflow.

The average value of the friction coefficient through the entire diffuser was approximately 50 per cent higher than that for fully developed turbulent flow in smooth pipes. Friction coefficients at the diffuser entrance were about three times the pipe values. In the middle of the diffuser, the friction coefficients agreed well with smooth-pipe values. R. C. Binder, USA

724. Seymour M. Bogdonoff, "NACA cascade data for the blade design of high-performance axial-flow compressors," *J. aero. Sci.*, Feb. 1948, vol. 15, pp. 89-96.

In this paper a short report on an interesting series of experimental investigations is given. Having despaired of the possibility of theoretically obtaining the data required in compressor blade design, the author obtained and describes the results of tests with both low-speed and high-speed two-dimensional cascade tunnels, and with a low-speed test blower. These results showed that the data obtained with the low-speed cascade tunnel are acceptable also for high speeds and for blades mounted on a rotor.

Furthermore, the low-speed tests have shown that it is possible to make use of highly loaded blades, so that the author concludes that with the loadings and Mach numbers shown possible by these tests, a performance comparable to the best existing axial-flow compressor could be obtained with half the number of stages. Unfortunately, the data given in this paper are too few and the conclusions insufficiently numerically specified to be sure about the importance of these results. As an example, the maximum lift coefficient admissible appears to be about 1.2, which is not far from those commonly used.

Gino Moretti, Italy

725. J. F. Runckel and R. S. Davey, "Pressure-distribution measurements on the rotating blades of a single-stage axial-flow compressor," *Nat. adv. Comm. Aero. Tech. Note No. 1189*, Feb. 1947, pp. 1–34.

The authors describe the results of an experimental program in which pressure distributions about the mean radius section of the rotating blades of a single-stage axial-flow compressor, operating at a blade Mach number of 0.35, were measured. The use of the NACA multicell rotating pressure transfer device represents a major aspect of the project. Pressure distributions previously obtained for similar stationary isolated airfoils were on the whole confirmed. However, the lift-curve slope was significantly lower than indicated by estimated values from theoretical calculations for comparable two-dimensional cascades. No systematic results are given, but only sufficient data to confirm the essential need for rotating-cascade data and to try out the experimental technique. Newman A. Hall, USA

726. A. G. Holms and R. D. Faldetta, "Effects of temperature distribution and elastic properties of materials on gas-turbine-disk stresses," *Nat. adv. Comm. Aero. Tech. Note No. 1334*, June 1947, pp. 1–22.

The determination of the influence of changes in the temperature distribution and in elastic properties, for a gas-turbine disk, is based on a report by S. S. Manson [*Nat. adv. Comm. Aero. Tech. Note No. 1279*, May 1947].

The present investigation indicates that: (1) It is advisable to minimize the temperature gradients, since severe temperature variations cause thermal stresses of considerable magnitude; (2) design calculations require accurate data on the elastic modulus and the coefficient of thermal expansion over the temperature ranges encountered, and when the variations of these properties are neglected inaccurate results are obtained; (3) the effect of changes in Poisson's ratio on the total stresses is small.

Sophocles J. Dokos, USA

Experimental Flow Equipment and Technique

(See also Revs. 676, 686, 714, 724, 725)

727. M. T. Hockman and R. E. Eisiminger, Jr., "The correlation of wind-tunnel and flight-test stability and control data for an SB2C-1 airplane," *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 5–17.

In order to determine the reliability of wind-tunnel tests on powered models in predicting stability and control characteristics, some comparable wind-tunnel and flight-test data for a Curtiss SB2C-1 airplane were analyzed to ascertain the degree of correlation between the two types of data. The data compared was for longitudinal stability, elevator effectiveness, longitudinal and directional control, and stick and pedal forces.

The methods used for the analysis of both wind-tunnel and flight-test data are discussed. The conditions under which both the wind-tunnel and flight-test data were obtained are clearly given, and the possible sources of error in each type of data are discussed in detail.

The conclusion is reached that the correlation for the particular airplane and model considered was satisfactory. It is pointed out, however, that further study of the correlation of such data for other airplanes is needed in order to determine to what degree such wind-tunnel data may be safely used for design purposes. J. S. Isenberg, USA

728. H. T. Epstein and L. U. Albers, "The effects of compressibility on the two-dimensional subsonic wind-tunnel constriction correction," *J. aero. Sci.*, Mar. 1948, vol. 15, pp. 144–150.

Tests of large models in wind tunnels require correction for constriction, or the effect of the finite cross-sectional area of the model in constricting and accelerating the flow. The constriction effect has been investigated for incompressible flow by conformal mapping methods and for compressible flow by the linear per-

turbation theories of Goldstein and Young, and Allen and Vincenti. In the present paper, relaxation methods are employed in the study of this effect in two-dimensional flow.

Calculations are reported for Kaplan's 10 per cent thick symmetrical section, both isolated and symmetrically placed between two walls, for ratios of chord to tunnel height of 1 and 2, and for flows at Mach numbers of 0 (incompressible flow) and 0.5. The calculation procedure is given in detail in an appendix. The incompressible velocities agreed well with those given by conformal mapping, and the compressible velocities for the free-stream case agreed well with those given by Kaplan.

Evaluation of the wall interference showed that the effect of compressibility, averaged linearly along the chord, agreed with the predictions of Goldstein and Young, and Allen and Vincenti. However, the variation along the chord is so great as to make questionable the validity of the use of these theories for the prediction of local effects. Stuart R. Brinkley, Jr., USA

729. **H. Ludwieg, "Drag corrections in high-speed wind tunnels,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1163*, July 1947, pp. 1–33 (transl. from *Zent. wiss. Berichtswesen Berlin Forschungsber.*, 1944).

For models in wind tunnels or open jets it is necessary to apply three different correction factors because of the finite size of the stream. These are the displacement correction factor, the dead-air displacement correction factor, and the dead-air pressure-gradient displacement correction factor.

The latter factor is due to the motion of the dead air or wake downstream from the model. The author has calculated this factor for incompressible flow and applied the results to compressible flow by means of the Prandtl analogy. The limitations of the derived correction factor are discussed, and it is compared graphically with the other two drag correction factors; it is shown to be of the same order of magnitude as they are, and even considerably larger in special cases. W. W. Hagerty, USA

Hydraulics; Transport of Solids; Cavitation

730. **Stefano Teofilato, "Measurement of shock phenomenon in high-speed water currents (Misure di fenomeni d'urto in corrente veloce),"** *Monogr. sci. Aero. Suppl. Tecn.*, Jan. 1947, no. 1, pp. 1–14.

The author begins with the computation of the relation between the direction of the oblique hydraulic jump and the deviation angle imposed on a torrential flow having a given Mach number.

In order to explain a discrepancy with his experiments, he analyzes the interpretation which must be given to the indications of a Pitot tube for a torrential flow, taking into account the formation of a shock wave ahead of the instrument.

In an appendix, a tentative calculation of the length of a normal jump does not seem to take sufficient account of the particular factors contributing to a shock wave in a liquid under free surface conditions. A. Craya, France

731. **Léopold Escande and Vladimir Goutkin, "Comparison of different methods of calculation applied to the surge tank at Bioge (Comparaison de diverses méthodes de calcul appliquées à la chambre d'équilibre complexe de Bioge),"** *C. R. Acad. Sci., Paris*, June 30, 1947, vol. 224, pp. 1808–1810.

Methods previously developed by the same authors for calculating the unsteady-state changes in level of a surge tank, feeding a turbine from two or more reservoirs, have been applied to the tank at Bioge. The results show that the change in level during the first oscillation can be satisfactorily calculated by an approximate graphical method in which the actual tank is replaced by one of uniform cross section. No comparison with experiment is given. B. G. Rightmire, USA

732. **L. Escande, "Graphical method for studying oscillations in a reservoir whose discharge canal receives flows from wells of small cross section (Méthode graphique pour l'étude des oscillations dans une chambre d'équilibre dont le canal d'amenée collecte des apports de débit par des puits de section négligeable),"** *Houille blanche*, July-Oct. 1947, vol 2, pp. 2–32.

In a previous paper the author studied the oscillations of the water surface in a reservoir due to influx from wells situated along the discharge canal, by means of the calculus of finite differences and a semigraphical method. In this paper he shows that, if these wells have negligible cross section, a simple graphical procedure may be applied in a large number of cases.

J. A. Lewis, USA

733. **M. S. Plesset and P. A. Shaffer, Jr., "Drag in cavitating flow,"** *Rev. mod. Phys.*, Jan. 1948, vol. 20, pp. 228–231.

The authors present a method of computing the cavity drag of a body of revolution based upon an extension of Riabouchinsky's theory of the cavity drag of a flat plate, and compare the calculated results with experimental data published by Reichardt. The history of the calculation of cavity drag is presented and the results of various theories are discussed briefly.

Riabouchinsky calculated the force on a flat plate by making use of the free streamline theory and introducing an image plate downstream of the "true" plate. The authors extend this method to the case of a wedge and obtain expressions for the velocity and pressure distribution over the wedge, and from these an equation for the coefficient of drag. Assuming that the pressure distribution for a cone is identical with that of the wedge, the coefficient of drag is computed for cones of various apex angles. The wedge theory is given in considerable detail, but no details are given for the case of the cone, the authors stating that these will be given elsewhere.

The agreement of the calculated values of drag coefficient and Reichardt's experimentally determined values is good except in the case of the cone of 14 deg angle. No explanation has been found for the deviations at this angle. Warren E. Wilson, USA

Marine Propulsion

(See also Revs. 584, 709)

734. **Herbert Wagner, "Planing of watercraft,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1139*, Apr. 1948, pp. 1–41 (transl. from *Jb. Schiffbautech.*, 1933, vol. 34).

This article presents a general discussion of the elements to be considered in determining the resistance and lift of boats which are operating in smooth water at speeds where the major portion of the weight is carried by planing action rather than displacement. It appears to be a physical discussion of a more mathematical article by the same author ["Über Stoss-und Gleitvorgänge an der Oberfläche von Flüssigkeiten," *Z. angew. Math. Mech.*, Aug. 1932].

The chief contribution of the author to the knowledge of this complex subject is that, by comparison with a comparable airfoil, he is able to analyze the effect of the spray set up by a planing surface and the effect of the width of the surface. His brief discussion of the planing of V-bottom surfaces and of impact of a

step and of a V-bottom boat is of some interest, although it would seem that his other article should be studied if quantitative results are desired. A pertinent discussion by F. Weinig covering further analysis of pressure distribution for larger planing angles and the formation of waves and backwash behind the planing surface is included. F. Everett Reed, USA

Lubrication; Wear

735. J. C. Livengood and Chapin Wallour, "A study of piston-ring friction," *Nat. adv. Comm. Aero. Tech. Note No. 1249*, Aug. 1947, pp. 1–48.

This paper discusses tests on a special engine equipped with a crosshead and an elastically mounted cylinder, by which it was possible to measure the friction forces between the piston rings and the cylinder wall during operation. Tests were run on a cast-iron piston ring against an SAE 4140 barrel, a cast-iron ring against a porous chrome-plated barrel, and a chrome-plated top ring against the SAE 4140 barrel. Data were also taken on the surface profiles of the barrel before, during, and after tests, showing the effect of run-in on the surface finish.

The following conclusions concerning piston-ring friction are drawn: (1) Friction decreased slightly with increasing running time, most of the change occurring during the first hour. (2) The cast-iron piston rings operating in an SAE 4140 barrel had the lowest friction, the cast-iron rings in a porous chrome barrel had the greatest, and the SAE 4140 barrel with one chrome top ring was intermediate. The differences were small. (3) Friction increased with engine speed. (4) Friction decreased with increased cylinder-jacket temperature. (5) Lowering the manifold pressure reduced friction. (6) The oil flow past the piston ring, from the cylinder head toward the crankcase, was much larger with a porous chrome-plated barrel with all cast-iron rings than with the other combinations.

Erle I. Shobert, II, USA

Dynamics of Meteorology and Oceanography

(See also Rev. 706)

736. Paul Queney, "On the theory of cyclones (Sur la théorie des cyclones)," *C. R. Acad. Sci., Paris*, Oct. 6, 1947, vol. 225, pp. 584–585.

The author states some of the results of his studies on the free oscillations in a rectilinear or zonal wind current, taking into account the effects of the earth's rotation and curvature, and of horizontal and vertical gradients of wind velocity and temperature. In a uniform current he finds long-period inertia waves of the type first discussed by Rossby. When the wind speed increases with altitude he finds the stability of the waves is increased if the temperature decreases toward the pole and decreased for the reversed gradient. The effect of the horizontal temperature gradient varies with the wave length of the disturbance and is a maximum for wave lengths of 2000–6000 km. The normal temperature gradient in middle latitudes must thus be considered as opposing the formation of cyclone waves.

H. J. Stewart, USA

737. K. F. Bowden, "Some observations of waves and other fluctuations in a tidal current," *Proc. roy. Soc. London, Ser. A*, Feb. 18, 1948, vol. 192, pp. 403–425.

Simultaneous measurements were conducted of the fluctuations in the speed of a tidal current, and of pressure. Both types of observations were carried out at various depths between surface and bottom. The periods investigated extend from 2 sec upward. Statistical analyses of the data involve, in addition to more elementary methods, serial correlation coefficients and periodograms. Results are compared to hydrodynamic theory.

Short-period fluctuations in current and pressure are associated with surface waves, though the large inherent variability of these waves prevents definite conclusions as to the validity of the theory. In addition, long-period fluctuations, of period from 30 sec upward, appear on the current records. It is suggested by the author that these may be related to turbulence. An interesting feature is the dependence of these long-period fluctuations on the mean current.

The pressure recorder used does not appear comparable to "differential-type" pressure recorders recently developed in England and the United States. The author's experience demonstrates the need for instrumental analysis of records.

A very useful feature of the paper is the appendix, in which s developed for the first time equations valid to the third approximation for the pressure and orbital velocities of waves of finite height in shallow water. Walter H. Munk, USA

738. Walter H. Munk, "A critical wind speed for air-sea boundary processes," *J. mar. Res.*, 1947, vol. 6, no. 3, pp. 203–218.

The variations with wind speed of a number of phenomena at the air-sea interface are shown to be discontinuous at a speed of 700 cm per sec. These phenomena include the occurrence of "whitecaps," the soaring of sea gulls, the resistance coefficient, and the evaporation coefficient.

From the Kelvin-Helmholtz theory of waves at a surface of discontinuity, it has been known that the wind must exceed about 660 cm per sec for the sea waves to be unstable. Waves can be formed by winds of only 110 cm per sec, but these are regarded as stable, not producing a hydrodynamically rough surface. The unstable wavelets occurring only when the wind exceeds about 660 cm per sec are regarded as the roughness elements which convert the sea from a hydrodynamical smooth, though not plane, surface. The character of turbulence in the air flow over the sea thus would also be transformed at the same wind speed as that at which the character of the sea surface changes. The growth of wind waves is considered to be accelerated by the roughening of the sea, which gives rise to additional tangential stresses. M. Neiburger, USA

739. K. Weekes and M. V. Wilkes, "Atmospheric oscillations and the resonance theory," *Proc. roy. Soc. London, Ser. A*, Dec. 23, 1947, vol. 192, pp. 80–99.

The authors study the oscillations of the atmospheric pressure as produced by tide-producing forces, using the theories of Taylor [*Proc. roy. Soc. London*, 1936, vol. 156, p. 318] and Pekeris [*Proc. roy. Soc. London*, 1937, vol. 158, p. 650]. The main point of the argument consists in pointing out: (1) That the oscillations of pressure are capable of being analyzed as plane waves traveling in a medium having a suitable refractive index; (2) that, as this refractive index is a function of the height, it can become imaginary between certain levels, and damped or undamped resonance is then made possible. When this is verified, periods of 8 and 12 hr must be possible for the free oscillations of the atmosphere, since these are the periods of the lunar and solar forces.

The problem, being strictly connected to the variation of temperature with the height, allows comparisons with measurements and hypotheses concerning this variation. Some possible models of temperature distributions which allow these free oscillations are theoretically established. In the simplest one, the

temperature maintains a minimum value between 15 and 25 km, attains a maximum at about 60 km, and finally, above 80 km, remains constant. This is in good agreement with the measured values. Other possible temperature distributions are also studied (the calculations being made with the aid of a differential analyzer), and interesting conclusions are drawn.

Gino Moretti, Italy

740. **O. G. Sutton, "The theoretical distribution of air-borne pollution from factory chimneys,"** *Quart. J. roy. met. Soc.*, July-Oct. 1947, vol. 73, pp. 426–436.

Equations are presented for the concentration at any position, and for the distance downwind to the point of maximum concentration at the ground, of pollution coming from a point source of constant strength at a distance h above the ground. The distance downwind to maximum concentration is proportional to $h^{2/(2-n)}$, if the wind speed is independent of height. Here n, a parameter with a value between zero and one which is related to the diffusing power of the turbulence, is determined by the momentum transfer indicated by the wind shear. For normal stability, the author finds $n = 1/4$.

The diffusion coefficients and other parameters were evaluated from data obtained at the Chemical Defense Experiment Station, Porton, England, and these values used to compute concentrations downstream. Tables are presented giving the concentrations and distance downwind to the point of maximum concentration at the ground, for various chimney heights.

The effect of stability is considered. Values of the parameters for various temperature lapse rates are given, and tables showing their influence on the intensity and position of maximum concentration at the ground are presented. The effect of an inversion is to increase the maximum concentration at the ground, and to increase the distance downwind at which it occurs.

The results are compared with Hewson's observational data at Trail, British Columbia. The correspondence is fair, although the author recognizes that computations based on experiments over a plain cannot validly be transferred to a mountain valley.

M. Neiburger, USA

Ballistics; Detonics (Explosions)

(See also Revs. 745, 753)

741. **J. A. Van Allen and H. P. Hitchcock, "Loss of spin of projectiles: Part I. Experimental method; Part II. Skin friction drag,"** *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 35–40.

Part I of this paper describes an experimental method used in determining the aerodynamic spin-resisting couple. The nose fuse is replaced by a small battery-powered radio-frequency oscillator coupled to an antenna which has a radiation pattern like that of a dipole. The axially asymmetric pattern results in an amplitude and polarization modulated radio-frequency signal in a receiver at a ground station. The periodicity of the signal depends upon the spin velocity, and an electronic counter is used for correlation with the time of flight.

In Part II, the results of firings of shell from 75 mm to 240 mm are discussed. The couple coefficient is tabulated and the skin friction drag estimated. Philip Rosenberg, USA

742. **William R. Sears, "On projectiles of minimum wave drag,"** *Quart. appl. Math.*, Jan. 1947, vol. 4, pp. 361–366.

This paper continues von Kármán's treatment of the wave resistance of slender bodies of revolution in supersonic flow by the linearized first-order theory. Von Kármán's work is reviewed. The body is considered to be generated by a distribution of sources along its axis. An integral formula is developed for the wave resistance in terms of the source distribution and is shown to be analogous to the formula for the induced drag of a finite wing in the Prandtl lifting line theory. The author uses Prandtl's methods to solve the wave-resistance equation by representing the source distribution with a sine series. The wave resistance is obtained as a function of the coefficients of the series.

The author next uses his solution to determine the shape having minimum wave resistance for a given length and volume. The type of solution proves fortunate because the minimal shape can be determined immediately from the solution by inspection. Explicit formulas are obtained for the wave resistance and the shape.

The author next considers the similar minimal problem for a body with a given length and caliber. Here the solution requires the methods of the variation calculus, and allowance is made for a singularity in the source function at the position of maximum cross section. The author again appeals to the analogy of the induced wing drag to determine the nature of the singularity. Explicit expressions are obtained for the source distribution and the wave resistance as functions of the length, the maximum cross section, and the position of the maximum cross section.

It is interesting to note that the shape giving minimum drag in both cases is symmetrical fore and aft.

A. C. Charters, USA

743. **Hermann Kurzweg, "Fundamental aerodynamic investigations for development of arrow-stabilized projectiles,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1175*, Dec. 1947, pp. 1–46 (transl. from *Dtsch. Akad. Luftfahrtforsch. Schr.*, no. 1059/43, 1943, p. 33).

Stabilization of projectiles by fins rather than by spinning is not a recent idea, but basic analyses and data have been lacking, especially for supersonic velocities. Wind-tunnel tests reported by the author not only demonstrate that fin stabilization is practicable at supersonic velocities, but also provide considerable quantitative data. Normal forces are preferable to drag forces for stabilization.

An aerodynamic analysis leads to recommendations as to optimum shape, size, and location of control surfaces. The difficulty of obtaining adequate control unless the fins project beyond the body of the projectile is explained, and it is shown that conclusions as to stability drawn from wind-tunnel tests on a small model are on the safe side. The author proposes a new design of finned projectile with sliding control surfaces.

Other investigations made in Germany are outlined, and it is brought out that the increase in range obtainable with a finned projectile as compared with a spinning projectile decreases rapidly as the diameter increases. C. W. Smith, USA

Thermodynamics

(See also Revs. 594, 681, 685, 755, 756)

744. **K. E. Grew, "Thermal diffusion in mixtures of the inert gases,"** *Proc. roy. Soc. London Ser. A*, May 1, 1947, vol. 189, pp. 402–414.

Experimental data are presented on thermal diffusion in all binary mixtures of the inert gases, except Kr-X and Rn mixtures, at temperatures between −180 and 400 C. The variation with temperature of the thermal diffusion ratio k_T (ratio of gradient of concentration of either gas to logarithmic gradient of tempera-

ture along diffusion path) is determined. The ratio R_T between k_T and $k_{T\infty}$ (theoretical value of k_T for molecules, of diameters deduced from viscosity data, which interact like rigid elastic spheres) is found to increase with temperature at low temperatures and to become constant at high temperatures. This constant value is first attained at temperatures which vary for different mixtures, but its magnitude is nearly the same for all mixtures, being approximately equal to the theoretical value for molecules which act as centers of an inverse 11th-power repulsive force.

The results are compared with previously published data which give mean values of R_T in the range 15 to 100 C for the same gas mixtures. There is reasonably good agreement except in the case of the He-Ne mixture, and repetition of the tests over a range of compositions of the mixture confirmed the present results.

The experimental results are compared with theoretical results for molecules which act as centers of inverse 9th-and 5th-power repulsive and attractive forces, respectively. The character of the dependence of R_T on temperature is found to be in reasonable agreement with the theory, although the theoretical values are uniformly low. The author suggests that a closer agreement between theory and experiment might be obtained if the exponents of the theoretical force fields were suitably chosen. It is further suggested that the exponents of molecular force fields may be determined by experimental determination of the variation of R_T with temperature. W. G. Cornell, USA

745. R. O. King, "The oxidation, ignition, and detonation of fuel vapors and gases," *Canad. J. Res. Sec. F*, Nov. 1947, vol. 25, pp. 326–342; Jan. 1948, vol. 26, pp. 36–56.

This paper is in two parts. The first part deals with the effect of small amounts of metallic "dopes" such as iron carbonyl or tetraethyl lead on the rate of oxidation of the liquid paraffins in engine fuel. It was found that the rate of oxidation was strongly affected by the flow pattern in the combustion chamber. This accounted for differences in behavior between laboratory experiments and engine tests.

The second part of the paper deals with the relation between the flow configuration, the nature of the exposed surfaces, and the reaction velocity, especially in narrow annular spaces of varied design. W. W. Hagerty, USA

746. W. A. Leary, E. S. Taylor, C. F. Taylor, and J. U. Jovellanos, "The effect of fuel composition, compression pressure, and fuel-air ratio on the compression-ignition characteristics of several fuels," *Nat. adv. Comm. Aero. Tech. Note No. 1470*, Mar. 1948, pp. 1–107.

The authors attempt to correlate the compression-ignition characteristics of a fresh charge of air-fuel mixture with the detonating tendencies of the last fraction of the charge in the engine. Correlation of results with actual engine test data of the fuels tested is poor, since the last fraction is not composed of the same molecular structure inducted. The method of test is not new, but is far more tedious and has proved less positive than actual engine tests. C. J. Vogt, USA

747. Manfred Schäfer, "Equations for adiabatic but rotational steady gas flows without friction," *Nat. adv. Comm. Aero. Tech. Memo. No. 1187*, Aug. 1947, pp. 1–23 (transl. from *Lehrstuhl tech. Mech. T. H. Dresden Arch.*, no. 14/1).

In this paper the equations for rotational steady gas flow of an ideal gas are derived on the basis of certain assumptions. The main assumptions are: (*a*) The flow is isoenergetic; in other words, the energy equation is valid with the same constants for the entire flow; (*b*) the gas behaves adiabatically, not during the compression shock itself, but before and after the shock.

The author essentially derives the basic equations and then gives their general application to two-dimensional and rotationally symmetrical flows, the final result being given in the form of a differential equation for the velocity field, the solution of which is not discussed. Application to practical problems would necessitate solutions of the equations presented in the paper.

Ahmed D. Kafadar, USA

Heat Transfer

(See also Rev. 744)

748. Pierre Vernotte, "Extension of Fourier's method to composite systems with resistance to heat flow between certain regions (Extension de la méthode de Fourier à l'étude des systèmes complexes dans lesquels certains milieux sont séparés par une résistance de passage)," *C. R. Acad. Sci., Paris*, May 19, 1947, vol. 224, pp. 1416–1418.

The author first explains the derivative of the known identity

$$m\int\int\int_V c\rho\varphi\psi\,d\tau \equiv \int\int\int_v \lambda\left(\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial x}+\frac{\partial\varphi}{\partial y}\frac{\partial\psi}{\partial y}+\frac{\partial\varphi}{\partial z}\frac{\partial\psi}{\partial z}\right)d\tau+\int\int_\Sigma h\varphi\psi\,d\sigma$$

upon which is based the general method of calculation of a Fourier expansion for the solution of a cooling problem. In the above expression c, ρ, and λ represent, respectively, the specific heat, density, and conductivity of each medium; h is the coefficient of heat transfer toward the exterior; φ is a continuous function of the co-ordinates such that $\varphi(x, y, z)e^{-mt}$ is a solution of the equation of heat conductors; ψ is an arbitrary continuous function; V is the volume of the system; Σ is the total exterior surface; and $d\tau$, $d\sigma$ are volume and surface elements, respectively.

In the case of contact resistance between bodies, the above identity does not hold, since there is a discontinuity. The author shows that for such a system the right-hand side of the identity contains the additional term

$$\int\int_{\Sigma'} \lambda^2 R\,\frac{\partial\varphi}{\partial n}\frac{\partial\psi}{\partial n}\,d\sigma$$

when Σ' represents the portions of the surface between two media with resistance R between them. It is stated that either the above identity or the modified form of it serves as a basis for Fourier calculations only by their symmetry in φ and ψ.

Ahmed D. Kafadar, USA

749. G. N. Kroozhilin, "Heat emission from a horizontal plate to a boiling liquid at free convection" (in Russian), *Notes Acad. Sci. USSR* (*Doklady Ak. Nauk SSSR*), Dec. 11, 1947, vol. 58, no. 8, pp. 1657–1660.

Heat transfer from a heated surface to boiling liquid is characterized by the value α of the heat-transfer coefficient, and by the the value of the "critical rate of heat input" q_{cr} at which the mode of boiling changes from bubbles to a film. These values are functions of physical constants, as well as of the roughness of the surface, the latter determining the number of centers of vaporization. The author points out that it is not possible at the present time to construct a quantitative theory for the in-

fluence of the roughness. He therefore limits his study to the calculation of the heat transfer as a function of the physical constants only, considering first the case of transfer by a single point of vaporization, and then the influence of the presence of a number of points on the over-all heat transfer. From test data of M. Jakob and W. Linke the author concludes that the thickness of the film and the velocity of rise of the bubbles may be neglected.

On the basis of the test data of M. Cichelli and C. Bonilla and of R. H. Braunlich, the relations between the physical factors which determine the heat transfer are given in the form of nondimensional equations. Transformation of these into dimensional equations gives

$$\alpha = 4.0 \frac{\lambda^{0.8}\gamma_1^{0.7}(\gamma_2 r)^{0.033} q^{0.7}}{\mu^{0.5}(\gamma_1 - \gamma_2)^{0.4} T_s^{0.37} c^{1/6}\sigma^{1/3}} \text{ cal per sq meter per hr per °C}$$

$$q_{cr} = 12 \times 10^6 \frac{\lambda^{0.5}(\gamma_1 - \gamma_2)^{7/8}(\gamma_2 r T_s)^{1/3}\sigma^{1/24}}{\gamma_1^{3/4} c^{1/6}} \text{ cal per sq meter per hr}$$

where λ, γ_1, γ_2, r, T_s, c, σ, μ are respectively the coefficient of heat transfer for the liquid, specific weights of liquid and vapor, heat of vaporization, absolute temperature of saturation, specific heat, surface tension, and kinematic viscosity.

The nondimensional formulas show that α is approximately proportional to $\theta^{0.07}$, and q_{cr} to $\theta^{-0.55}$, where θ is the angle of wetting of the surface by the liquid. The author points out that these results are valid only when $\theta<90$ deg at boiling. For $\theta>90$ deg, for all rates of heat input greater than that corresponding to pure convection, the mode of boiling by film holds exclusively. D. Jacovleff, Belgium

750. J. C. Jaeger and Martha Clarke, "Numerical results for some problems on conduction of heat in slabs with various surface conditions," *Phil. Mag.*, July 1947, vol. 38, pp. 504–515.

The authors solve problems on the conduction of heat in slabs with various boundary conditions. Although solutions of most of these problems have been published, the authors found that some of them were inadequate, and many seemed to be of questionable accuracy. The authors, therefore, have systematically and independently recalculated the whole group of problems, and have included several new cases. The problems involve a one-dimensional solid region, the "slab" $(0 < x < l)$ with various boundary conditions at $x = 0$ and $x = l$. The boundary conditions prescribed involve temperature, flow of heat, radiation, and contact with well-stirred fluid.

The paper discusses the method of attack and gives the fundamental functions involved (eight functions of two dimensionless parameters, the time parameter and the parameter for the outer conductivity) as well as graphs of these functions plotted against the arc tangent of the outer conductivity parameter, for various values of the time parameter. The fundamental functions are separately applied to problems involving the various boundary conditions mentioned above. It is pointed out that solutions of a number of other problems can be deduced immediately from those given in the paper. Ahmed D. Kafadar, USA

751. P. L. Kapitza, "Theoretical and empirical expressions for heat transfer in two-dimensional turbulent flow" (in English), *C. R. Acad. Sci. URSS*, Mar. 10, 1947, vol. 55, no. 7, pp. 591–597.

Equations for heat-transfer correlation based on heat transfer-momentum transfer analogy are reviewed. The equations, developed by Reynolds, Prandtl and Taylor, and von Kármán, are all based on the assumed existence of combinations of three separate and distinct regions—purely laminar layer, transition region, and turbulent region. In each region different laws of transfer apply.

The author writes the heat-transfer and friction equations in terms of a mixing length

$$Q = -(k + c\rho\overline{vl})\partial T/\partial y \quad [12]$$

$$\tau = -(\mu + \rho\overline{vl})\partial U/\partial y \quad [13]$$

where the mixing factor $\overline{vl}$ denotes time averaged product of the fluctuations of velocity v perpendicular to the wall and the mixing length l, Q is rate of heat transfer per unit area, τ shear stress, k thermal conductivity, μ coefficient of viscosity, c specific heat, ρ density, T temperature, U stream velocity, and y distance perpendicular to the wall.

There being no experimental evidence of clearly defined boundaries between the assumed laminar, transition, and turbulent regions, the author suggests that $\overline{vl}$ be expressed as a single continuous function of y. Such a function should make the velocity profile satisfy the following three conditions: (1) At the wall

$$\left|\frac{\partial U}{\partial y}\right|_{y=0} = \frac{U_\tau^2}{\nu} \quad [19]$$

where the friction velocity U_τ is determined from the shear stress at the wall $\tau_0 = \rho U_\tau^2$, and ν is the kinematic viscosity. (2) In the region very near the wall

$$\frac{\partial U}{\partial y} = \frac{U_\tau^2}{\nu} f\left(\frac{U_\tau y}{\nu}\right) \quad [21]$$

as suggested by Prandtl and confirmed by Nikuradse. (3) At a distance from the wall where U is measurable, a "power law" variation may be assumed

$$\frac{U}{U_\tau} \sim \left(\frac{U_\tau y}{\nu}\right)^n \quad [22]$$

The author suggests the following possible form of the function which would satisfy the foregoing three conditions

$$\overline{vl} = \frac{U_\tau^2}{\nu} \frac{Cy^2}{1 + B\left(\frac{U_\tau y}{\nu}\right)^{1-n}} \quad [23]$$

where B, C, n are constants.

Combining Equations [12], [13], and [23] with the Blasius expression for friction factor γ in terms of Reynolds number Re

$$\gamma = 0.0791\ \mathrm{Re}^{-0.25} \quad [29]$$

the author arrives at the heat-transfer correlation equation

$$\frac{\alpha}{cG} = 0.0396\ \mathrm{Re}^{-0.25}\ \mathrm{Pr}^{-0.57} \quad [30]$$

where α is film coefficient of heat transfer, G is mass velocity, and Pr is Prandtl number. This equation is quite similar to the generally accepted empirical correlation equation. The main assumption underlying it seems to be the mixture-factor expression, Equation [23], whose justification may require a more thorough experimental investigation of heat transfer and velocity phenomena close to the wall. Warren M. Rohsenow, USA

752. C. J. Tranter, "Heat flow in an infinite medium heated by a cylinder," *Phil. Mag.*, Feb. 1947, vol. 38, pp. 131–134.

The author gives a solution of the heat-conduction problem in which an infinite medium is heated by a long circular cylinder of a

different material. Thermal diffusivities and the thermal conductivities of the cylinder and the medium being assumed, the author solves the boundary-value problem by the use of the Laplace transform. Although similar problems have been treated before, the author presents an approximate solution which is shown to give good results for short times, as compared to the exact solution. A numerical table shows the comparison.

Ahmed D. Kafadar, USA

753. G. Comenetz, "Continuous heating of a hollow cylinder," *Quart. appl. Math.*, Jan. 1948, vol. 5, pp. 503–510.

The author gives an application of the solution of the heat-conduction equation for hollow cylinders obtained by Carslaw and Jaeger [*Proc. Lond. math. Soc.*, 1939, vol. 46, p. 361]. Carslaw and Jaeger's derivations give rise to slowly convergent series, which are summed by the author in finite terms. He considers the cases of the heat-input rate through the inner surface as a linear function, and as a quadratic function, of time. The outer surface of the cylinder is assumed to be either maintained at zero temperature or thermally insulated.

Important applications of this solution are made in determining the temperature distribution in the barrel of a gun in which the heat input decreases almost linearly with time, as under continuous firing conditions. The case of intermittent heat input to the bore surface of a gun is also briefly treated.

The solutions given in this paper will have many other applications, such as heating of a tubular furnace, a chimney, and the insulation on a wire carrying an electric current. Although various approximate methods have been developed for this purpose, the author's method should give better results.

Ahmed D. Kafadar, USA

754. A. G. Walters, "A problem on the conduction of heat," *Phil. Mag.*, Jan. 1947, vol. 38, pp. 70–78.

The author makes use of series expansions analogous to those used by Kneser [*Math. Ann.*, 1907, vol. 63, p. 511] and Sommerfeld [*Jber. dtsch. MatVer.*, 1913, vol. 12, p. 309] in solving problems in heat conduction with complex boundary conditions. The problem considered here concerns a long tubular body A, which is contained in a long steel tube with which it is concentric. There is an air space between the body and the tube. The long cylindrical cavity along the center of the tubular body contains still air. Further, there is a thin refractory coating on the inside surface of the steel tube.

The whole body is heated to a uniform temperature and then is allowed to cool in air which may or may not be at rest. The problem is treated mathematically without giving numerical applications. In an appendix properties of the Bessel function used are given. Proof of the series expansion used in the solution of the problem is also included. Ahmed D. Kafadar, USA

755. J. B. Ubbink, "Thermal conductivity of gaseous helium," *Physica Hague*, Dec. 1947, vol. 13, pp. 629–634.

Experimental data for the heat conductivity λ of He gas reported in the literature are compared with theoretical values calculated as functions of the viscosity η. The temperature and pressure dependence of λ is also considered.

According to the kinetic theory of gases $\lambda = f\eta c_v$, f being a constant between 2.500 and 2.522, and c_v the specific heat at constant volume. Correcting the available values of η for He on the basis of the best available values of η for air, and correlating them with experimental λ values for temperatures from 273 K down to 1.64 K, the author concludes that $f = 2.5$ is in good agreement with the available data, but voices the need for more accurate absolute determinations of η.

Verification of classical theory for the temperature dependence of λ is unimportant since for low temperatures the deBroglie wave length is much larger than the kinetic diameter. Quantum mechanical values calculated by deBoer agree reasonably well with experimental data.

The theory of the pressure dependence of λ must consider the deviation of the gas from the ideal state (Enskog), and the ratio of the deBroglie wave length to the distance of the molecules. Best agreement between experimental and theoretical values is credited to deBoer, who uses the concept of "potential field" diameter, in place of "hard" classical diameter.

Hans F. Winterkorn, USA

756. J. B. Ubbink, "Pressure dependence of the coefficient of heat conductivity for the gases helium and hydrogen at low temperatures," *Physica Hague*, Dec. 1947, vol. 13, pp. 659–668.

An apparatus is described for the measurement of the coefficient of heat conductivity λ of gases as a function of pressure. This involves a 0.1-mm gas layer which can be filled with gas at any desired pressure. Heat produced by a heating wire is transmitted through (*a*) metallic conduction around the edges of the layer, and (*b*) by gaseous conduction across the gas layer. λ is calculated on the basis of the double-transfer mechanism indicated, the metallic conduction factor being evaluated from experimental results obtained with completely evacuated gas chamber. Special experimental precautions are necessary, and the material for the apparatus must be properly chosen.

At 20 K, λ for He was found to be independent of the pressure, within the limits of experimental accuracy. For H_2, however, at 24 K, λ was a function of the pressure; at pressures of 0 cm and 100 cm Hg, λ for H_2 was respectively $4.51.10^{-5}$ and $4.56.10^{-5}$ cal per cm per sec per deg.

At 4.20 K, λ for He was found to be a function of the pressure; variation of the pressure from 0 to 100 cm Hg caused a variation of from 2.092×10^{-5} to 2.206×10^{-5} cal per cm per sec per deg., which is an increase of 5.2 per cent for 100 cm Hg. The experimental data showed that even at high densities λ remained proportional to the density. Hans F. Winterkorn, USA

757. Y. S. Touloukian, G. A. Hawkins, and M. Jakob, "Heat transfer by free convection from heated vertical surfaces to liquids" *Trans. Amer. Soc. mech. Engrs.*, Jan. 1948, vol. 70, pp. 13–18.

The authors' experiments reported here deal with the study of heat transfer by natural convection from vertical cylindrical surfaces to water and ethylene glycol. The results can be correlated within the laminar range $2 \times 10^8 < N_{Gr} \times N_{Pr} < 4 \times 10^{10}$ by the expression

$$N_{Nu} = 0.726\ (N_{Gr} \times N_{Pr})^{1/4}$$

Within the turbulent range, for $4 \times 10^{10} < N_{Gr} \times N_{Pr} < 9 \times 10^{11}$, the correlation is best represented by

$$N_{Nu} = 0.0674\ [N_{Gr}\ (N_{Pr})^{1.29}]^{1/3}$$

These correlations represent the experimental data with a mean deviation of 5.6 per cent and 3.5 per cent for the laminar and turbulent ranges, respectively. The ranges of some of the important variables involved in the tests are given in Table 1 of the text. The experimental setup and test procedures are described. Charts are given to show the correlations.

Ahmed D. Kafadar, USA